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RESEARCH LOADED SHELLS OF REVOLUTION SUPPORTED BY RIBS OF DIFFERENT SHAPES

The usage of superelement method for composite shells of rotation enabled to work out an effective algorithm of calculation of their stress-strain state. For the element of ring-type the local stiffness matrix has been made, thus giving the possibility to observe shell-type ribs as well as ring-type ones, which cross-section sizes are smaller than their radius. A number of calculations have been made to compare the impact of the rib form upon the tense state of shells.

Key words: *shell rotation, superelement, stiffness matrix, the stress-strain state, rib.*

To calculate the stress state of closed shell of rotation at axis-symmetrical loading we use the system of differential equations [1], [5]:

$$\begin{cases} \frac{d}{ds}\left(rN_S\right)-\sin\varphi N_\theta-\frac{r}{r_1}Q_S+rq_S=0 \\ \frac{d}{ds}\left(rQ_S\right)+\frac{r}{r_1}N_S+\cos\varphi N_\theta-rq_r=0 \\ \frac{d}{ds}\left(rM_S\right)-\sin\varphi M_\theta+rQ_S=0, \end{cases} \quad (1)$$

where $N_S=D_N(\varepsilon_S+\nu\varepsilon_\theta)$, $M_S=D_M(\chi_S+\nu\chi_\theta)$, $D_N=Eh/(1-\nu^2)$, $N_\theta=D_N(\nu\varepsilon_S+\varepsilon_\theta)$, $M_\theta=D_M(\nu\chi_S+\chi_\theta)$, $D_M=Eh^3/(12\cdot(1-\nu^2))$,

$$\varepsilon_S=\frac{dU}{ds}+\frac{W}{r_1},\ \varepsilon_\theta=\frac{\sin\varphi}{r}U+\frac{\cos\varphi}{r}W, \quad (2)$$

$$\chi_S=\frac{d\theta_S}{ds},\quad \chi_\theta=\frac{\sin\varphi}{r}\theta_S,\quad \theta_S=-\frac{dW}{ds}+\frac{U}{r_1}.$$

In formula (2) E is modulus of elasticity, ν is Poisson's ratio, h is thickness of shell, r_1 is curvature radius of meridian line.

Let's introduce unknown values $y_1=N_S$, $y_2=Q_S$, $y_3=M_S$, $y_4=U$, $y_5=W$, $y_6=-\theta_S$.

By this the system (1) is brought to the normal system of ordinary differential equations relative to unknown functions, that will become:

$$\begin{cases} \frac{dy_1}{ds}=-\frac{\sin\varphi}{r}(1-\nu)y_1+\frac{1}{r_1}y_2+\frac{\sin^2\varphi}{r^2}D_N\left(1-\nu^2\right)y_4+ \\ \quad +\frac{\sin\varphi\cos\varphi}{r^2}D_N\left(1-\nu^2\right)y_5-q_S \\ \frac{dy_2}{ds}=\left(-\frac{1}{r_1}-\frac{\nu\cos\varphi}{r}\right)y_1-\frac{\sin\varphi}{r}y_2-\frac{\cos\varphi\sin\varphi}{r^2}D_N\left(1-\nu^2\right)y_4- \\ \quad -\frac{\cos^2\varphi}{r^2}D_N\left(1-\nu^2\right)y_5+q_r \\ \frac{dy_3}{ds}=-y_2-\frac{\sin\varphi}{r}(1-\nu)y_3-\frac{\sin^2\varphi}{r^2}D_M\left(1-\nu^2\right)y_6 \\ \frac{dy_4}{ds}=\frac{1}{D_N}y_1-\nu\frac{\sin\varphi}{r}y_4-\left(\frac{1}{r_1}+\frac{\nu\cos\varphi}{r}\right)y_5 \\ \frac{dy_5}{ds}=\frac{1}{r_1}y_4+y_6 \\ \frac{dy_6}{ds}=-\frac{1}{D_M}y_3-\nu\frac{\sin\varphi}{r}y_6. \end{cases} \quad (3)$$

The middle surface of the shell should be in parametric form:

$$\begin{cases} x=x(t) \\ y=y(t) \end{cases},\quad (t_0\le t\le t_1). \quad (4)$$

Functions, included into the system (3), are determined according to the formula

$$\sin\varphi=\frac{x'}{\sqrt{x'^2+y'^2}};\quad \cos\varphi=\frac{y'}{\sqrt{x'^2+y'^2}};$$

$$\frac{1}{r_1}=\frac{x''y'-y''x'}{\left(x'^2+y'^2\right)^{\frac{3}{2}}};\quad dS=\sqrt{x'^2+y'^2}\,dt. \quad (5)$$

Let's use the method of superelements based on the finite element method, but allows observation of more extensive shells. To build local stiffness matrix of superelements we use Godunov's method of differential equation system solution, that has higher accuracy of solution [2]. It was reachable due to ortogonalization of intermediate solutions.

To make the local stiffness matrix of superelement, it is necessary to solve a series of two-pointed edging problems for a homogeneous system

$$\frac{1}{B}\frac{dy}{dt} = Ay \,. \tag{6}$$

$$\begin{matrix} 1) \begin{cases} y_4(t_0)=1 \\ y_5(t_0)=0 \\ y_6(t_0)=0 \\ y_4(t_1)=0 \\ y_5(t_1)=0 \\ y_6(t_1)=0, \end{cases} & 2) \begin{cases} y_4(t_0)=0 \\ y_5(t_0)=1 \\ y_6(t_0)=0 \\ y_4(t_1)=0 \\ y_5(t_1)=0 \\ y_6(t_1)=0, \end{cases} & 3) \begin{cases} y_4(t_0)=0 \\ y_5(t_0)=0 \\ y_6(t_0)=0 \\ y_4(t_1)=0 \\ y_5(t_1)=0 \\ y_6(t_1)=1. \end{cases} \end{matrix} \tag{7}$$

Here $y^T = (y_1 \cdots y_6)$, $B = \sqrt{x'^2 + y'^2}$, A – is a linear system operator (3).

The first column of the local stiffness matrix $\overline{k^e}$ will be full as a vector of solution for the first variant of initial data, the second as for the second variant of initial data, … , the sixth column will correspond to the 6th variant of problems (6), (7).

By solving non-homogeneous problems

$$\frac{1}{B}\frac{dy}{dt} = Ay + q \tag{8}$$

at zero boundary conditions $y_4(t_0)=0$, $y_5(t_0)=0$, $y_6(t_0)=0$, $y_4(t_1)=0$, $y_5(t_1)=0$, $y_6(t_1)=0$ the vector of forces, acting at the left and right edges in the local system of these edges coordinates is found:

$$\overline{f^e} = \begin{pmatrix} f_1^e & f_2^e & f_3^e & f_4^e & f_5^e & f_6^e \end{pmatrix}^T, \tag{9}$$

where $q^T = (-q_S \quad q_r \quad 0 \quad 0 \quad 0 \quad 0)$ is an acting loading.

To make the global stiffness matrix and vectors of force loading we should build the local stiffness matrices and force in junctions for global coordinate system. The transformation is made according to the formula:

$$\begin{aligned} k^e &= T^T \cdot \overline{k^e} \cdot T \cdot \theta \\ f^e &= -T^T \cdot \overline{f^e} \cdot \theta \,, \end{aligned} \tag{10}$$

where

$$T = \begin{pmatrix} \cos\varphi_0 & \sin\varphi_0 & & & & \\ -\sin\varphi_0 & \cos\varphi_0 & & & 0 & \\ & & 1 & & & \\ & & & \cos\varphi_1 & \sin\varphi_1 & \\ & 0 & & -\sin\varphi_1 & \cos\varphi_1 & \\ & & & & & 1 \end{pmatrix},$$

$$\theta = 2\pi \begin{pmatrix} -R_0 & & & & & \\ & R_0 & & & 0 & \\ & & R_0 & & & \\ & & & R_1 & & \\ & 0 & & & -R_1 & \\ & & & & & -R_1 \end{pmatrix}. \tag{11}$$

Here $\cos\varphi_0$, $\sin\varphi_0$, $\cos\varphi_1$, $\sin\varphi_1$, $R_0 = y(t_0)$, $R_1 = y(t_1)$ – are functions, formed according to the formula (4), (5) for initial and final focal points of superelement.

Matrix θ is formed as consequence of consideration of superelement reaction at the edges. The system of linear algebraical equations for finding of the displacements in focal points of shell superelement is:

$$K \cdot \delta = f \,, \tag{12}$$

where $K = \sum_e {}^{©} k^e$ is a global stiffness matrix,

$f = \sum_e {}^{©} f^e$ is a global vector of force,

$\delta^T = (U_1, W_1, -\theta_{S1}, \ldots, U_k, W_k, -\theta_{Sk})$ – are the global displacements in focal points. Sign $\sum^{©}$ means the ensemble process used in the finite element method [3].

This methods has allowed to consider ribs as shell constructions. In case ribs sizes are small compared to their radiuses, it is possible to use local focal matrices of stiffness for consideration of their impact. These matrices were built with the use of the results [4] at the observation of *z*-shaped rib. In the global coordinate system they are:

$$k_r = 2\pi \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{EF}{R} & -\dfrac{aFE}{R} \\ 0 & -\dfrac{aFE}{R} & \dfrac{EJ_y + a^2 FE}{R} \end{pmatrix}. \tag{13}$$

where E is modulus of elasticity, F is area of section, R is radius of the shell in the focal point, J_y is a moment of inertia with respect to an $0y$ axis, going through the center of section mass, a is deviation of focal point from the center of rib mass.

Local matrix k_r is added to the matrix K by method of ensemble.

Presence of symmetry in the local stiffness matrices ensures the symmetry of global matrix and we can use matrix operations with banded matrices for solution (12). After solution there will be displacements in the global system of coordinates in the focal points of superelements. To find the stress state of superelement from the global solution δ the displacements are chosen at its edges δ^e and then they are recalculated into the displacements in the local system of coordinates

$$\overline{\delta^e} = T \cdot \delta^e . \qquad (14)$$

Then the corresponding non-homogeneous two-pointed edging problem (6) is solved with the following initial data:

Then the corresponding non-homogeneous two-pointed edging problem (6) is solved with the following initial data:

$$y_4(t_1) = \overline{\delta_4^e}, \quad y_5(t_1) = \overline{\delta_5^e}, \quad y_6(t_1) = \overline{\delta_6^e}, \qquad (15)$$

where $\overline{\delta_1^e},...,\overline{\delta_6^e}$ are $\overline{\delta^e}$ matrix elements.

A set of applied programs for personal computer was worked out for the investigated methods.

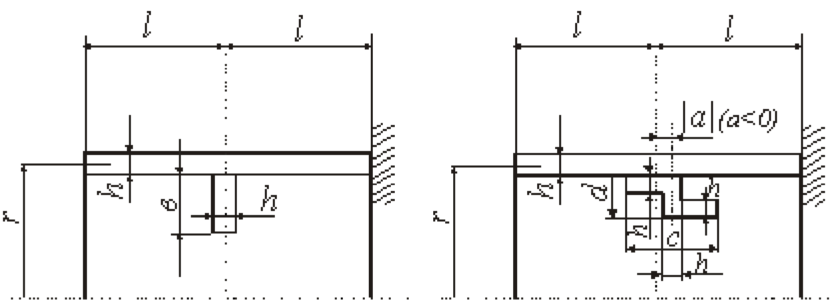
There were made calculations of stress-strain state of cylindrical shells for comparative analyses: 1. Without ribs, 2. With plate ring element of shell-type, 3. With rectangular rib of ring-type, that has characteristics of the case No2, 4. Z-shaped rib. The type of ribs and their geometrical characteristics are given in Figure 1. Cylindrical shell and ribs are made of material with $E = 2.1 \cdot 10^5$ MPa – modulus of elasticity, $n = 0.3$ – is Poisson's ratio. Thickness of shell $h = 0.03$ m. The left edge of the shell is free, the right is fixed. The shell is loaded with normal pressure $q_r = 200$ H/m².

Given above variants are calculated for the data:

$l = 0.5$m, $h = 0.03$m, $a = -0.0275$m, $b = 0.2$m, $c = 0.14$m, $d = 0.12$m.

Z-shaped rib has the same area of cross-section as ribs 2, 3.

The results of the W deflection calculations, Q_S shearing force and M_S curving moment depending on the point distance of the middle surface from the left butt-edge of the shell are given in fig.2.



a *b*

Fig. 1. Geometrical characteristics of cylindrical shell with ribs: *a* is ring shaped rib (cases 2,3), *b* is z-shaped rib (4)

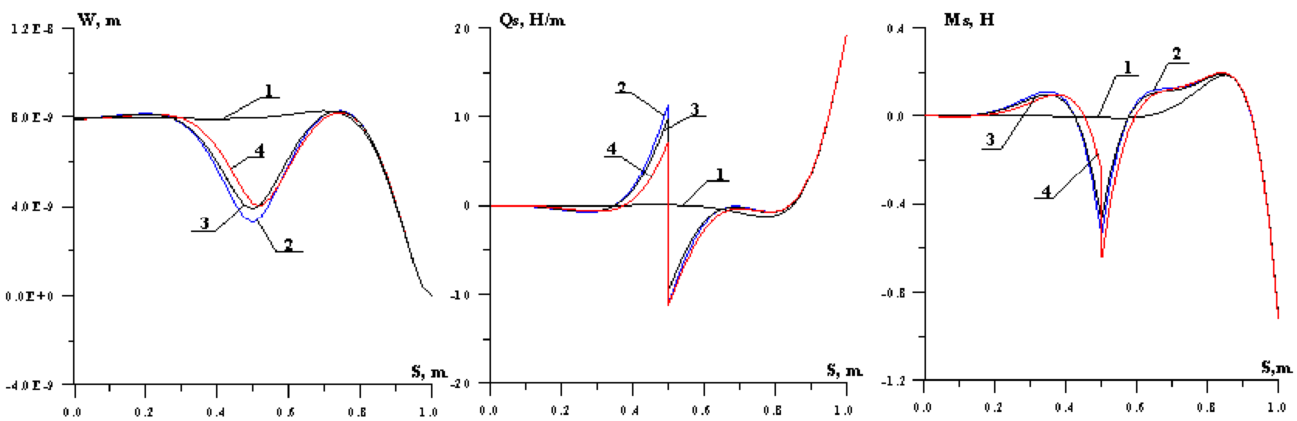


Fig. 2. The results of calculation of W(s), Q_S(s) and M_S(s) for different types of ribs

It is visible from comparison that the results of the calculations for the shell construction (2) and rectangular rib of ring-type (3) coincide. Thus there is an opportunity to calculate the shells with the ribs of the arbitrary section, for example, case 4, that are closely attached to the shell surface.

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Левицька Т.І., Пожуєва І.С. Дослідження навантажених оболонок обертання підкріплених шпангоутами різної форми

Застосування методу суперелементів для складених оболонок обертання дозволило розробити ефективний алгоритм знаходження їх напружено-деформованого стану. Побудова локальної матриці жорсткості для елемента кільцевого типу дало змогу розглядати як шпангоути оболонкового типу, так і шпангоути кільцевого типу, розміри поперечного перерізу яких малі порівняно з їх радіусом. Проведено ряд розрахунків для порівняння впливу форми шпангоутів на напружений стан оболонок.

Ключові слова: *оболонки обертання, суперелемент, матриця жорсткості, напружено-деформований стан, шпангоут.*

Левицкая Т.И., Пожуева И.С. Исследование нагруженных оболочек вращения подкрепленных шпангоутами разной формы

Применение метода суперэлементов для составных оболочек вращения позволило разработать эффективный алгоритм нахождения их напряженно-деформированного состояния. Построение локальной матрицы жесткости для элемента кольцевого типа дало возможность рассматривать как шпангоуты оболочечного типа, так и шпангоуты кольцевого типа, размеры поперечного сечения которых малы по сравнению с их радиусом. Проведен ряд расчетов для сравнения влияния формы шпангоутов на напряженное состояние оболочек.

Ключевые слова: *оболочки вращения, суперэлемент, матрица жесткости, напряженно-деформированное состояние, шпангоут.*