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THE EFFECT OF A MOVING LOAD ON A THREE-LAYER CYLINDRICAL SHELL WITH A TRANSVERSAL ISOTROPIC FILLER

Purpose. *To extend the approach previously proposed by the authors on the application of exact equations of elasticity theory to problems of dynamics of three-layer cylindrical shells with isotropic filler to one of the possible cases of anisotropy of the middle layer material, namely the situation when the filler is transversely isotropic. To obtain accurate formulas and, based on them, to construct a picture of the stress-strain state in such a composite structure when moving along the outer surface at a constant normal (radial) load speed.*

Research methods. *A mathematical model of the dynamics of a three-layer cylindrical shell has been constructed, where the motion of the supporting layers is described by the equations of thin shell theory, and for a transversely isotropic filler, the dynamic equations of the theory of elasticity of an anisotropic medium in general form are used. When considering the problem in a stationary setting, Galilean transformation is applied, after which the integral Fourier transform in complex form is applied to all sought and given values in the moving coordinate system. To calculate non-proper Fourier integrals, quadrature formulas based on the Filon method for integrating rapidly oscillating functions were developed, which made it possible to efficiently obtain numerical results with a predetermined accuracy.*

Results. *Based on the constructed model, the problem of a moving load that causes a stationary stress-strain state of a layered cylindrical shell under various conditions on the surfaces of the joint between the filler and the supporting layers is considered. In this case, the contact is considered both rigid and sliding, but the lag of the shells from the filler is excluded. The difficulties that arise when solving the equations of motion of a transversely isotropic filler are overcome by introducing a special method using undefined coefficients of potential functions. For all possible boundary conditions, the results are obtained in the form of non-special improper integrals, which are calculated using special quadrature formulas. The distribution patterns of displacements and stresses along both the length and thickness of the filler are shown, a comparison with the results for the corresponding isotropic filler is made, and a mechanical analysis of the results is performed.*

Scientific novelty*. For the first time in such a formulation, when the behaviour of the filler is described by exact equations of the dynamics of an elastic anisotropic body, a solution to the stationary dynamic problem for a three-layer cylindrical shell has been obtained. A comparison was made with the results previously obtained for the case of isotropic filler. A special technique was used to introduce potential functions to find displacements and stresses in the dynamic equations for transversely isotropic materials. Important partial boundary conditions at the boundaries of layer contacts were considered.*

Practical value. *The results obtained with this approach can be used as reference values when constructing simplified models of the dynamic behaviour of three-layer cylindrical shells, in particular those that take into account the anisotropy of the filler. Examples of such materials include so-called ribbed sound-insulating materials.*

Key words*: Layered shells, transversely isotropic medium, integral transformation, potential functions, boundary conditions, displacement, stress.*

Introduction

Dynamic problems for three-layer plates and shells are of considerable practical interest due to the fact that such structural elements are extremely widely used, in particular in aircraft and rocket construction, maritime transport, construction and other industries. At the same time, historically, at the beginning of the development of

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the theory of such systems, various simplified approaches were proposed, based on hypotheses about the behaviour of a lighter and softer filler relative to the load-bearing layers, which made it possible to obtain equations for describing the behaviour of the entire three-layer package which were no more complex than similar equations for a single-layer shell or plate. It is clear that the models would be accurate if each of the layers obeyed (was described by) the dynamic equations of elasticity theory, but in this case, the calculation algorithm became much more complex and could not be used effectively enough in engineering calculations. Therefore, this approach is usually used for comparison with the results obtained using the simplified models mentioned above, and examples of solutions using the exact approach and the comparisons are given in monographs [1-4]. As numerous examples have shown, the most effective approach in terms of the complexity-accuracy ratio, especially in so-called stationary dynamic problems, is the one where the motion of the load-bearing layers (which are relatively very thin in relation to the thickness of the entire layered plate or shell) is described by the equations of the theory of single-layer shells, which are based on the Kirchhoff-Lyaev or Timoshenko hypotheses, and the dynamic equations of elasticity theory are used for the filler. In particular, monographs [1–2] present a large number of stationary and non-stationary problems considered in this formulation, comparisons with such solutions and with simplified approaches, but in all the problems presented, the filler was considered isotropic (homogeneous or with variable mechanical characteristics in thickness). This paper considers one of the problems of this class in order to show how the approaches previously developed by the authors can be extended to the case where the filler material has different properties depending on the direction, i.e. is significantly anisotropic, in particular in the case of a cylindrical three-layer shell, which is a transversely isotropic body.

Purpose of the work

A stationary dynamic problem is considered concerning the reaction of a three-layer infinitely long cylindrical shell to the movement of an axisymmetric radial load along its outer surface at a constant subcritical speed. The aim of the work is to extend the previously proposed approaches based on Galilean transformation, the application of integral transformations, and the use of numerical algorithms developed by the authors to reverse these transformations to the case of a transversely isotropic filler. At the same time, the main focus is on the analytical solution of dynamic equations for transversely isotropic filler material by introducing potential functions in a special way, which made it possible to obtain the transformants of displacements and stresses in the image space in the form of combinations of Bessel functions. The results obtained in this work can be used to construct simpler engineering models for shells with transversely isotropic fillers.

Material and research methods

An infinitely long three-layer cylindrical shell is considered, in the general case of an asymmetrical structure in terms of thickness, i.e., one in which the thickness and mechanical characteristics of the load-bearing layers may be different, and the filler, which is significantly lighter and softer than the shell materials, is transversely isotropic. A self-balanced radial load moves along the outer surface of the shell at a constant subcritical and pre-seismic velocity. It is necessary to determine the stress-strain state at arbitrary points of the load-bearing layers and filler and to evaluate the influence of the anisotropy of the filler material by comparing it with similar results for isotropic material.

The mathematical model of the problem is constructed as follows. We will describe the motion relative to thin load-bearing layers using the most well-known equations in shell theory and more accurate equations of the Timoshenko type, which, incidentally, do not follow from the equations of elasticity theory by simplification, but are an intuitive discovery of an outstanding Ukrainian mechanic, or by simpler equations based on Kirchhoff-Leva's hypotheses and derived from the equations of elasticity theory. In the first case, for the axisymmetric problem considered here, the equations of motion of the skins are written as follows [5, 11, 13]

$$\frac{\partial^2 u}{\partial x^2}+\frac{\nu_k}{a_k}\frac{\partial w}{\partial x}=\rho_k\frac{1-\nu_k}{2G_k}\frac{\partial^2 u}{\partial t^2}-\frac{1-\nu_k}{2G_k h_k}q_{xk}$$

$$\beta^2\left(\frac{\partial^2 w}{\partial x^2}-\frac{\partial \alpha}{\partial x}\right)-\frac{2}{1-\nu_k}\left(\frac{\nu_k}{a_k}\frac{\partial u}{\partial x}+\frac{w}{a_k^2}\right)=$$

$$=\rho_k\frac{1-\nu_k}{2G_k}\frac{\partial^2 w}{\partial t^2}-\frac{1-\nu_k}{2G_k h_k}\left(q_{rk}+p_k\right)$$

$$\beta^2\left(\frac{\partial w}{\partial x}-\alpha\right)+\frac{h_k^2}{6\left(1-\nu_k\right)}\frac{\partial^2 \alpha}{\partial x^2}=\rho_k\frac{h_k^2}{3G_k}\frac{\partial^2 \alpha}{\partial t^2}. \quad (1)$$

Here, the index k = 1 refers to the inner load-bearing layer, and k = 2 refers to the outer layer, u, w are the displacements of the points of the middle surface of the corresponding shell in the axial and radial directions, α is the angle of rotation of the normal to the middle surface, Gk, ρk are the shear modulus and density of the materials of the corresponding shells, hk, ak are the thicknesses and radii of the middle surfaces of the load-bearing layers, β2 is the Timoshenko coefficient, qrk, qxk are the radial and axial reactions from the filler on the motion of the load-bearing layers , p1 = 0, p2 – intensity of external moving load.

For Kirchhoff-Lyaev equations, we will have the following [7, 10, 12]

$$\frac{\partial^2 u}{\partial x^2}+\frac{\nu_k}{a_k}\frac{\partial w}{\partial x}=\rho_k\frac{1-\nu_k}{2G_k}\frac{\partial^2 u}{\partial t^2}-\frac{1-\nu_k}{2G_k h_k}q_{xk},$$

$$\frac{\nu_k}{a_k}\frac{\partial u}{\partial x}+\frac{h_k^2}{12}\frac{\partial^4 w}{\partial x^4}+\frac{w}{a_k^2}=\rho_k\frac{1-\nu_k}{2G_k}\frac{\partial^2 w}{\partial t^2}-\frac{1-\nu_k}{2G_k h_k}\left(q_{rk}+p_k\right). \quad (2)$$

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Considering the filler to be transversely isotropic, we write the dependencies between stresses and strains in the general spatial case in the following form

$$\sigma_{xx} = c_{11}\varepsilon_{xx} + c_{12}\varepsilon_{yy} + c_{13}\varepsilon_{zz},$$
$$\sigma_{yy} = c_{12}\varepsilon_{xx} + c_{11}\varepsilon_{yy} + c_{13}\varepsilon_{zz},$$
$$\sigma_{zz} = c_{13}(\varepsilon_{xx} + \varepsilon_{yy}) + c_{33}\varepsilon_{zz},$$
$$\sigma_{xy} = \frac{1}{2}(c_{11} - c_{12})\varepsilon_{xy}, \quad \sigma_{yz} = c_{44}\varepsilon_{yz}, \quad \sigma_{zx} = c_{44}\varepsilon_{zx}. \quad (3)$$

In practice, the following technical constants are usually used: E, E' – Young's moduli for tension – compression in the direction of the plane of isotropy and, respectively, in the direction normal to this plane, ν – Poisson's ratio, which characterises transverse compression in the plane of isotropy when stretched in this plane, ν' – the same characteristic when stretched in the direction normal to the plane of isotropy, G = E / 2(1+ν), G' = E' / 2(1+ν') – shear moduli for the plane of isotropy and any plane perpendicular to it.

In this case, we obtain the necessary dependencies for further use in an obvious way [6]

$$c_{11} = \frac{E\left(E' - \nu'^2 E\right)}{(1+\nu)\left[(1-\nu)E' - 2\nu'^2 E\right]},$$
$$c_{12} = \frac{E\left(\nu E' + \nu'^2 E\right)}{(1+\nu)\left[(1-\nu)E' - 2\nu'^2 E\right]},$$
$$c_{13} = \frac{\nu' E E'}{(1-\nu)E' - 2\nu'^2 E},$$
$$c_{33} = \frac{(1-\nu)E'^2}{(1-\nu)E' - 2\nu'^2 E}, \quad c_{44} = G'. \quad (4)$$

First, we write down the dynamic equations of elasticity theory in stresses for the filler, which in a cylindrical coordinate system and for an axisymmetric problem have the following form [15]

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \sigma_{rx}}{\partial r} + \frac{1}{r}\sigma_{rx} = \rho \frac{\partial^2 u_x}{\partial t^2},$$
$$\frac{\partial \sigma_{rr}}{\partial r} + \frac{\partial \sigma_{rx}}{\partial x} + \frac{1}{r}(\sigma_{rr} - \sigma_{\theta\theta}) = \rho \frac{\partial^2 u_r}{\partial t^2}. \quad (5)$$

and then, rewriting the dependencies (3) in a cylindrical coordinate system and using the Cauchy relations according to which

$$\varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{xx} = \frac{\partial u_x}{\partial x}, \quad \varepsilon_{\theta\theta} = \frac{u_r}{r}, \quad (6)$$
$$\varepsilon_{xr} = \frac{\partial u_r}{\partial x} + \frac{\partial u_x}{\partial r},$$

we arrive at the equations of motion of a transverse isotropic filler in displacements

$$c_{11}\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) + c_{44}\frac{\partial^2 u_r}{\partial x^2} + (c_{13} + c_{44})\frac{\partial^2 u_x}{\partial r \partial x} = \rho \frac{\partial^2 u_r}{\partial t^2}$$
$$c_{44}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial r}\right) + c_{33}\frac{\partial^2 u_x}{\partial x^2} + (c_{13} + c_{44})\frac{\partial}{\partial x}\left(\frac{\partial u_r}{\partial r} + \frac{u_r}{r}\right) = \rho \frac{\partial^2 u_x}{\partial t^2}. \quad (7)$$

Due to the fact that the thicknesses of the bearing layers are relatively small, it is assumed that the filler contacts each of the shells along their middle surfaces, and then the boundary conditions for the system of partial differential equations (7) are written as follows:

if the contact is considered to be sliding but lagging is excluded, then

$$r = a_k \quad \sigma_{rx} = 0, u_r = w, \sigma_{rr} = \begin{cases} -q_{r1} & (r = a_1) \\ q_{r2} & (r = a_2) \end{cases}. \quad (8)$$

considering the contact to be rigid, the boundary conditions are written as follows

at $r = a_k$ $u_r = w, u_x = u, \sigma_{rx} = -q_{xk}, \sigma_{rr} = \begin{cases} -q_{r1} & (r = a_1) \\ q_{r2} & (r = a_2) \end{cases}$. (9)

Note that since the problem is considered in the so-called steady-state formulation, there are no initial conditions for the system (7).

Since in this problem the load moves along the shell at a constant speed *c*, we apply Galilean transformation to all equations of the problem, according to which a moving coordinate system is introduced by the formulas

$$r' = r, \quad \eta = \frac{x - ct}{a_2}. \quad (10)$$

Now, in the coordinate system according to (10), the stress-strain state of our structure remains unchanged over time (the values do not depend on the variable *t*), but changes when the load velocity *c* changes.

Let us move to the moving coordinate system in equations (1), (2) and (7) according to formulas (10), then instead of (1) and (2) we will have ordinary differential equations with variable η, and (7) will become equations in partial derivatives with variables η and *r* = r'

$$\nu_k \frac{a_2}{a_k}\frac{dw}{d\eta} + \left(1 - \frac{\rho_k c^2}{2G_k}(1-\nu_k)\right)\frac{d^2u}{d\eta^2} = -\frac{(1-\nu_k)a_2}{2G_k h_k}q_{xk},$$
$$\left(\beta^2 - \frac{\rho_k c^2}{2G_k}(1-\nu_k)\right)\frac{d^2 w}{d\eta^2} - \frac{2}{1-\nu_k}\left(\frac{\nu_k}{a_k}\frac{du}{d\eta} + \frac{w}{a_k}\right) - \beta^2 a_2 \frac{d\alpha}{d\eta} = -\frac{(1-\nu_k)a_2}{2G_k h_k}(q_{rk} + p_k),$$
$$\beta^2\left(\frac{1}{a_2}\frac{dw}{d\eta} - \alpha\right) + \frac{h_k^2}{a_2^2}\left(\frac{1}{6(1-\nu_k)} - \frac{\rho_k c^2}{3G_k}\right)\frac{d^2\alpha}{d\eta^2} = 0. \quad (11)$$

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System (2) takes the following form

$$\left(1-\frac{(1-\nu_k)\rho_k c^2}{2G_k}\right)\frac{d^2u}{d\eta^2}+\nu_k\frac{a_2}{a_k}\frac{dw}{d\eta}=-\frac{(1-\nu_k)a_2}{2G_k h_k}q_{xk}$$

$$\nu_k\frac{a_2}{a_k}\frac{du}{d\eta}+\frac{h_k^2}{12a_2^2}\frac{d^4w}{d\eta^4}+\frac{wa_2}{a_k}-\frac{(1-\nu_k)\rho_k c^2}{2G_k}\frac{d^2w}{d\eta^2}= \qquad (12)$$
$$=-\frac{(1-\nu_k)a_2}{2G_k h_k}(q_{rk}+p_k)$$

The equation of motion of a transverse isotropic filler in a moving coordinate system is written as follows

$$c_{11}\frac{\partial}{\partial r}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)+\frac{1}{a_2^2}\left(c_{44}-\rho c^2\right)\frac{\partial^2 u_r}{\partial\eta^2}+$$
$$+\frac{1}{a}(c_{13}+c_{44})\frac{\partial^2 u_x}{\partial r\partial\eta}=0$$
$$c_{44}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_x}{\partial\eta}\right)+\frac{1}{a_2^2}\left(c_{33}-\rho c^2\right)\frac{\partial^2 u_x}{\partial\eta^2}+ \qquad (13)$$
$$+\frac{1}{a}(c_{13}+c_{44})\frac{\partial}{\partial\eta}\left(\frac{\partial u_r}{\partial r}+\frac{u_r}{r}\right)=0$$

Since the functions u, w, α, u_x , u_r together with their derivatives tend to zero at $|\eta|\to\infty$, then we apply to equations (11)–(13) and boundary conditions (8), (9) a complex integral Fourier transform with respect to the variable η

$$\overline{w}=\int_{-\infty}^{+\infty}we^{-i\xi\eta}d\eta,\quad w=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\overline{w}e^{i\xi\eta}d\xi. \qquad (14)$$

Then, in the image space, instead of (11) and (12) we will have systems of algebraic equations, and instead of (13) we will have a system of ordinary differential equations with respect to the transformants of the sought quantities

$$i\xi\frac{a_2}{a_k}\nu_k\overline{w}-\left(1-\frac{\rho_k c^2}{2G_k}(1-\nu_k)\right)\xi^2\overline{u}=-\frac{(1-\nu_k)a_2}{2G_k h_k}\overline{q}_{xk}$$
$$-\left(\beta^2-\frac{\rho_k c^2}{2G_k}(1-\nu_k)\right)\xi^2\overline{w}-\frac{2}{1-\nu_k}\left(i\xi\frac{\nu_k}{a_k}\overline{u}+\frac{\overline{w}}{a_k}\right)-$$
$$-i\xi\beta^2 a_2\overline{\alpha}=-\frac{(1-\nu_k)a_2}{2G_k h_k}(\overline{q}_{rk}+\overline{p}_k)$$
$$\beta^2\left(i\xi\frac{\overline{w}}{a_2}-\overline{\alpha}\right)-\frac{h_k^2}{a_2^2}\xi^2\left(\frac{1}{6(1-\nu_k)}-\frac{\rho_k c^2}{3G_k}\right)\overline{\alpha}=0. \qquad (15)$$

Equation (12) in the image space looks like this

$$i\xi\nu_k\frac{a_2}{a_k}\overline{u}+\frac{h_k^2}{12a_2^2}\xi^4\overline{w}+\frac{a_2}{a_k}\overline{w}+\frac{(1-\nu_k)\rho_k c^2}{2G_k}\xi^2\overline{w}=,$$
$$=-\frac{(1-\nu_k)a_2}{2G_k h_k}(\overline{q}_{rk}+\overline{p}_k)$$
$$-\left(1-\frac{(1-\nu_k)\rho_k c^2}{2G_k}\right)\xi^2\overline{u}+i\xi\nu_k\frac{a_2}{a_k}\overline{w}=-\frac{(1-\nu_k)a_2}{2G_k h_k}\overline{q}_{xk}. \qquad (16)$$

Before translating system (13) into the image space, it is advisable to introduce potential functions according to the following formulas

$$u_r=\frac{\partial}{\partial r}(\varphi+\psi),$$
$$u_x=\frac{1}{a_2}\frac{\partial}{\partial\eta}(k_1\varphi+k_2\psi). \qquad (17)$$

If we substitute expressions (17) into system (13), we obtain the following partial derivative equations for determining functions φ and ψ

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{x_1}{a_2^2}\frac{\partial^2}{\partial\eta^2}\right)\varphi=0.$$

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{x_2}{a_2^2}\frac{\partial^2}{\partial\eta^2}\right)\psi=0. \qquad (18)$$

where x_1 , x_2 are roots of the following quadratic equation that are not equal to each other

$$c_{11}c_{44}x^2-\left[\left(c_{44}-\rho c^2\right)c_{44}+\left(c_{33}-\rho c^2\right)c_{11}-(c_{13}+c_{44})^2\right]x+ \qquad (19)$$
$$+\left(c_{33}-\rho c^2\right)\left(c_{44}-\rho c^2\right)=0$$

and the unknown constants k_1 and k_2 are determined from the following relation

$$k(c_{13}+c_{44})+\left(c_{44}-\rho c^2\right)=c_{11}x. \qquad (20)$$

Note that in the case of an isotropic body, when the following conditions are satisfied E = E', ν = ν', G = G' equations (19) and (20) take the following form

$$x^2-\left(m^2+m_s^2\right)x+m^2m_s^2=0. \qquad (21)$$
$$k=2(1-\nu)x-(1-2\nu)m_s^2. \qquad (22)$$

Here

$$m^2=1-\frac{c^2}{c_p^2},\quad m_s^2=1-\frac{c^2}{c_s^2},$$

where c_p, c_s are the propagation velocities of tensile-compressive and shear waves in an isotropic medium

$$c_p=\sqrt{\frac{2G(1-\nu)}{\rho(1-2\nu)}},\quad c_s=\sqrt{\frac{G}{\rho}}.$$

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From equations (21) and (22), we find $x_1 = m^2$, $x_2 = m_s^2$, $k_1 = 1$, $k_2 = m_s^2$ and formula (18) becomes wave equations for an isotropic body.

The expressions for the components of the stress state of a transversely isotropic filler through potential functions, which are necessary for further calculations, are written as follows

$$\frac{\sigma_{rr}}{c_{44}} = -\frac{\beta + k_1}{a_2^2}\frac{\partial^2 \varphi}{\partial \eta^2} - \frac{\beta + k_2}{a_2^2}\frac{\partial^2 \psi}{\partial \eta^2} - \frac{c_{11} - c_{12}}{c_{44}}\frac{1}{r}\frac{\partial}{\partial r}(\varphi + \psi)$$

$$\frac{\sigma_{rx}}{c_{44}} = \frac{1 + k_1}{a_2}\frac{\partial^2 \varphi}{\partial r \partial \eta} + \frac{1 + k_2}{a_2}\frac{\partial^2 \psi}{\partial r \partial \eta}, \quad \beta = 1 - \frac{\rho c^2}{c_{44}}. \qquad (23)$$

In the image space, equations (18) take the following form

$$\frac{d^2\overline{\varphi}}{dr^2} + \frac{1}{r}\frac{d\overline{\varphi}}{dr} - \frac{x_1 \xi^2}{a_2^2}\overline{\varphi} = 0,$$

$$\frac{d^2\overline{\psi}}{dr^2} + \frac{1}{r}\frac{d\overline{\psi}}{dr} - \frac{x_2 \xi^2}{a_2^2}\overline{\psi} = 0. \qquad (24)$$

Each of the equations (24) is a Bessel equation, the solutions of which, depending on the values of x_1 and $x_{(2)}$, will be Bessel functions of various kinds. In particular, for load velocities that are less than the velocity of shear waves in the corresponding isotropic medium and less than the critical velocity, the roots x_1 and x_2 of equation (19) are real and positive numbers. Then the solutions of equations (24) take the following form

$$\overline{\varphi}(r,\xi) = A_1 K_0\left(\frac{e_1 \xi r}{a_2}\right) + A_2 I_0\left(\frac{e_1 \xi r}{a_2}\right),$$

$$\overline{\psi}(r,\xi) = A_3 K_0\left(\frac{e_2 \xi r}{a_2}\right) + A_4 I_0\left(\frac{e_2 \xi r}{a_2}\right), \qquad (25)$$

where $e_1 = \sqrt{x_1}$, $e_2 = \sqrt{x_2}$, $I_n(x)$, $K_n(x)$ are Bessel functions of the first and second kind from the imaginary argument [14].

Applying the Fourier transform to formulas (17) and (23), and then substituting expressions (25) into the transformed formulas, we find the transforms of displacements and stresses at all points of a transversely isotropic filler, in particular, we will have the following formulas

$$\overline{u}_r = -\frac{\xi}{a_2}\left[e_1 K_1(e_1 \xi r_*)A_1 + e_1 I_1(e_1 \xi r_*)A_2 + e_2 K_1(e_2 \xi r_*)A_3 + e_2 I_1(e_2 \xi r_*)A_4\right], \quad r_* = \frac{r}{a_2},$$

$$\frac{\overline{\sigma}_{rr}}{c_{44}} = \frac{\xi^2}{a_2^2}\left[(\beta + k_1)K_0(e_1 \xi r_*)A_1 + (\beta + k_1)I_0(e_1 \xi r_*)A_2 + (\beta + k_2)K_0(e_2 \xi r_*)A_3 + (\beta + k_2)I_0(e_2 \xi r_*)A_4\right] +$$

$$+ \frac{c_{11} - c_{12}}{c_{44}}\frac{\xi}{r_* a_2}\left[e_1 K_1(e_1 \xi r_*)A_1 + e_1 I_1(e_1 \xi r_*)A_2 + e_2 K_1(e_2 \xi r_*)A_3 + e_1 I_1(e_2 \xi r_*)A_4\right]. \qquad (26)$$

In the image space of equations (15) and (16), which are systems of algebraic equations, we use them to obtain dependencies between the transformants of loads transmitted to the outer and inner surfaces of the filler, and the displacements of the shells, which, according to conditions (8) and (9), coincide with the displacements at the boundaries of the filler. At the same time, if the contact between the skins and the filler is considered to be sliding, then in (15) and (16) we must set $\overline{q}_{xk} = 0$ and express $\overline{q}_{rk}$ only through $\overline{w}$. If, however, the contact is considered to be rigid, then $\overline{q}_{xk}$ and $\overline{q}_{rk}$ are found to be dependent simultaneously on $\overline{w}$ and $\overline{u}$. Based on these considerations, we find that in the case of a rigid contact with system (16), the above dependencies are written as follows

$$\overline{q}_{xk} = \frac{2G_k}{1 - \nu_k}\chi_k^2\left[\left(1 - \frac{1 - \nu_k}{3}c_{0k}^2\right)\xi^2 \overline{u} - i\xi \nu_k \varphi(k)\overline{w}\right]$$

$$\overline{q}_{rk} = -\overline{p}_k - \frac{2G_k}{1 - \nu_k}\chi_k^2\left[\left(\frac{\chi_k^2 \xi^4}{12} + \varphi^2(k) - \frac{1 - \nu_k}{3}c_{0k}^2 \xi^2\right)\overline{w} + i\xi \nu_k \varphi(k)\overline{u}\right] \qquad (27)$$

and in the case of sliding contact, we have the following

$$\overline{q}_{rk} = -\overline{p}_k - \frac{2G_k}{1 - \nu_k}\chi_k^2\left[\frac{\chi_k^2 \xi^4}{12} + \varphi^2(k)\left(1 - \frac{3\nu_k^2}{3 - (1 - \nu_k)c_{0k}^2}\right) - \frac{1 - \nu_k}{3}c_{0k}^2 \xi^2\right]\overline{w}. \qquad (28)$$

where the following designations are introduced

$$\varphi(k) = \begin{cases} \dfrac{1}{\varepsilon_1} & (k = 1) \\ 1 & (k = 2) \end{cases}, \quad \varepsilon_1 = 1 - \chi(1 + \chi_s),$$

$$\chi = \frac{h_2}{a_2}, \quad \chi_s = \frac{a_2 - a_1}{h_2} \quad c_0 = \sqrt{\frac{3\rho}{2G}} \cdot c.$$

The formulas for these dependencies have a similar form when using equations (15), but first we must exclude $\overline{\alpha}$ from the second and third equations and deal with a system similar to system (16).

If we now substitute expressions (27)–(28), as well as formulas of the form (26) for the transforms of displacements and stresses in the boundary conditions (18) or (19) recorded using the integral Fourier transform, we obtain a system of algebraic equations for determining the

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functions $A_1(\xi)\div A_4(\xi)$, the solution of which by Cramer's method will be written as follows

$$A_1=\frac{1}{G}\frac{a_2^2A_{41}}{\xi K_1(e_1\xi)det\|a_{ij}\|},\ A_2=-\frac{1}{G}\frac{a_2^2A_{42}}{\xi I_1(e_1\xi)det\|a_{ij}\|}$$

$$A_3=-\frac{i}{G}\frac{a_2^2A_{43}}{e_2\xi^2K_1(e_2\xi)det\|a_{ij}\|},$$

$$A_4=\frac{i}{G}\frac{a_2^2A_{44}}{e_2\xi^2I_1(e_2\xi)det\|a_{ij}\|}, \tag{29}$$

where the elements of the determinant $det\|a_{ij}\|$ in the case of sliding contact are calculated using the following formulas

$$\begin{aligned}
&a_{11}=2e_1s_1,\ a_{12}=-2e_1s_4,\ a_{13}=n_1s_7,\ a_{14}=n_1s_{10}\\
&a_{21}=2e_1,\ a_{22}=-2e_1,\ a_{23}=n_1,\ a_{24}=n_1,\\
&a_{31}=n_1\xi s_2+2e_1t_1s_1,\ a_{32}=n_1\xi s_5-2e_1t_1s_4, \quad (30)\\
&a_{33}=2(e_2\xi s_8+t_1s_7),\ a_{34}=-2(e_2\xi s_{11}-t_1s_{10}),\\
&a_{41}=n_1\xi s_3+2e_1t_2,\ a_{42}=n_1\xi s_6-2e_1t_2,\\
&a_{43}=2(e_2\xi s_{10}+t_2),\ a_{44}=-2(e_2\xi s_{12}-t_2),\\
&n_1=1+e_2^2,\ n_2=\frac{\chi\gamma}{1-\nu},\\
&t_1=1/\varepsilon_1,\ t_2=1-n_2n_4,\\
&\gamma=\frac{G_2}{G_1},\ \rho^*=\frac{\rho_2}{\rho},\\
&n_3=\frac{\chi^2\xi^4}{12}+\frac{1}{\varepsilon_1^2}\left(1-\frac{3\nu_k^2}{(1-\nu_k)c_0^2}\right)-\frac{1-\nu_k}{3}c_0^2\xi^2,\\
&s_1=\frac{K_1(e_1\xi\varepsilon_1)}{K_1(e_1\xi)},\ s_2=\frac{K_0(e_1\xi\varepsilon_1)}{K_1(e_1\xi)},\quad s_3=\frac{K_0(e_1\xi)}{K_1(e_1\xi)},
\end{aligned}$$

s_4– s_6 are obtained from s_1– s_3 by replacing the functions $K_n(x)$ with the functions $I_n(x)$, s_7– s_9 from s_1– s_3, and s_{10}– s_{12} from s_4 – s_6 by replacing e_1 with e_2, n_4 is found from n_3 when ε_1 = 1, and A_{4j} – minors of elements.

In the case of rigid contact (9), the functions A_k are calculated using formulas (29), if $det\|a_{ij}\|$ is replaced with $det\|b_{ij}\|$, A_{4j} with B_{4j}, and the elements of the determinants are found using the following formulas

$$\begin{aligned}
&b_{11}=2e_1(1-n_8)s_1-t_3s_2,\ b_{12}=-2e_1(1-n_8)s_4-t_3s_5,\\
&b_{13}=(n_1-2n_8)s_7-t_4s_8,\ b_{14}=(n_1-2n_8)s_{10}+t_4s_{11},\\
&b_{21}=2e_1(1-\nu n_2)-t_3s_3,\ b_{22}=-2e_1(1-\nu n_2)-t_3s_6,\\
&b_{23}=n_1-2\nu n_2-t_4s_9,\qquad b_{24}=n_1-2\nu n_2+t_4s_{12}\\
&b_{31}=2e_1n_9s_1+\xi(n_1+2n_8)s_2\\
&b_{32}=-2e_1n_9s_4+\xi(n_1+2n_8)s_5\\
&b_{33}=2n_9s_7+2e_2\xi(1+n_8)s_8,\\
&b_{34}=2n_9s_{10}-2e_2\xi(1+n_8)s_{11},\\
&b_{41}=2e_1n_{10}+\xi(n_1-2\nu n_2)s_3,\\
&b_{42}=-2e_1n_{10}+\xi(n_1-2\nu n_2)s_6,\\
&b_{43}=2n_{10}+2e_2\xi(1-\nu n_2)s_9,\\
&b_{44}=2n_{10}-2e_2\xi(1-\nu n_2)s_{12},\\
&n_5=\frac{\chi^2\xi^4}{12}+\frac{1}{\varepsilon_1^2}-\frac{1-\nu}{3}c_0^2\xi^2,\\
&n_7=1-\frac{1-\nu}{3}c_0^2,\ n_8=\frac{\nu}{\varepsilon_1}n_2,\\
&n_9=\frac{1}{\varepsilon_1}+n_2n_5\ \ n_{10}=1-n_2n_6\\
&t_3=2n_2n_7,
\end{aligned}$$

where n_6 is obtained from n_5 at ε_1 = 1.

After finding $A_1 \div A_4$ using formulas (29), we find the transformants of displacements and stresses using expressions (26). As a result, we arrive at the following dependencies

$$\frac{\bar{u}_r}{h_2}=-\frac{\nu(\xi,r_*)}{\chi G\,det\|a_{ij}\|}, \tag{32}$$

$$\nu(\xi,r_*)=e_1s_{14}A_{41}+e_2s_{16}A_{42}+s_{18}A_{43}-s_{20}A_{44}, \tag{33}$$

$$\bar{\sigma}_{rr}=\frac{\varepsilon(\xi,r_*)}{det\|a_{ij}\|}, \tag{34}$$

$$\varepsilon(\xi,r_*)=\left(n_1\xi s_{13}+\frac{2e_1}{r_*}s_{14}\right)A_{41}-\left(n_1\xi s_{16}-\frac{2e_1}{r_*}s_{16}\right)A_{42}+$$
$$+2\left(e_2\xi s_{17}+\frac{1}{r_*}s_{18}\right)A_{43}+2\left(e_2\xi s_{19}-\frac{1}{r_*}s_{20}\right)A_{44}, \tag{35}$$

$$s_{13}=\frac{K_0(e_1\xi r_*)}{K_1(e_1\xi)},\ s_{14}=\frac{K_1(e_1\xi r_*)}{K_1(e_1\xi)},\quad r_*=\frac{r}{a_2},$$

s_{15} – s_{16} are obtained from s_{13} – s_{14} by replacing the functions K_n with the functions I_n, s_{17} – s_{18} from s_{13} – s_{14}, and s_{19} – s_{20} from s_{15} – s_{16} by replacing e_1 with e_2.

As an example, calculations are performed for a ring load moving along the outer shell at a speed *of c*

$$p(x,t)=p_0\delta(x-ct), \tag{36}$$

where δ (x) is the Dirac delta function.

After applying the inverse Fourier transform in the space of originals, we obtain the following improper integrals for finding displacements and stresses at arbitrary points of a three-layer shell

$$\frac{2Gu_r}{p_0}=-\frac{2\gamma}{\pi}\int_0^\infty\frac{\nu(\xi,r_*)cos(\xi\eta)}{det\|a_{ij}\|}d\xi, \tag{37}$$

$$\frac{\sigma_{rr}a_2}{p_0}=\frac{1}{\pi}\int_0^\infty\frac{\varepsilon(\xi,r_*)cos(\xi\eta)}{det\|a_{ij}\|}d\xi. \tag{38}$$

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From the results obtained, we can obtain solutions for some boundary cases. In particular, if we assume that the inner surface of the filler does not contact the supporting layer, we obtain a problem about a shell with a hollow filler on the inner surface of which there are no stresses, then at $r = a_1$, we have the following boundary conditions

$$\sigma_{rr} = 0, \quad \sigma_{rx} = 0. \tag{39}$$

The solution is given by formulas (37) and (38) if some elements of the determinant are replaced as follows

$$a_{31} = n_1\xi s_2 + 2e_1\frac{s_1}{\varepsilon_1}, \quad a_{32} = n_1\xi s_5 - 2e_1\frac{s_4}{\varepsilon_1},$$
$$a_{33} = 2\left(e_2\xi s_8 + \frac{s_7}{\varepsilon_1}\right), \quad a_{34} = -2\left(e_2\xi s_{11} - \frac{s_{10}}{\varepsilon_1}\right). \tag{40}$$

Under conditions (39) and rigid contact between the outer shell and the filler in formulas (31), taking into account (40), the following substitutions must be made

$$b_{1j} = a_{1j}, \quad b_{3j} = a_{3j} \quad (j = 1, \;,4). \tag{41}$$

All other elements of the determinants $det\|a_{ij}\|$ and $det\|b_{ij}\|$ remain unchanged.

If the inner shell is considered to be absolutely rigid, then in the case of sliding contact between such a shell and the filler, the following conditions must be satisfied

when $r = a_1 \quad u_r = 0, \quad \sigma_{rx} = 0$ (42)

and in the case of rigid contact, this condition is written as follows

when $r = a_1 \quad u_x = 0, \quad u_r = 0$ (43)

and in the latter case, the elements in the determinant $det\|b_{ij}\|$ must be replaced as follows

$$b_{11} = -s_2, \quad b_{12} = -s_5, \quad b_{13} = -e_2 s_8, \quad b_{14} = e_2 s_{11},$$
$$b_{31} = es_1, \quad b_{32} = -es_4, \quad b_{33} = s_7, \quad b_{34} = s_{10}. \tag{44}$$

In all cases, to obtain the final results, it is necessary to calculate the improper Fourier integrals according to the formulas of the form (37), (38) and, since, as can be seen from the above calculations, the subintegral functions are very complex, these integrals cannot be calculated analytically, i.e., the answers cannot be obtained in the form of combinations of elementary and even special tabulated functions. In this regard, we have developed a special programme that takes into account the fact that, due to the presence of cosines, we are dealing with highly oscillating subintegral functions. Therefore, we applied a special method proposed by Filon for constructing quadrature formulas, which allowed us to obtain numerical results with any desired accuracy in an optimally short period of time on a personal computer.

As an example, calculations were performed for the following parameter values: $E/E' = 1.3$; $\nu = 0.3$; $\nu' = 0.2$; $\rho/\rho' = 1.5$; $h_1 = h_2$; $a_2/a_1 = 1.2$; $\chi = h_2/a_2 = 0.004$; $E_1 = E_2$; $\rho_1 = \rho_2$; $\gamma = E_1/E = 125$, $\rho^* = \rho_1/\rho = 12.5$. For comparison, the results for the corresponding isotropic filler are also given, where $E' = E$, $\nu' = \nu$, $\rho' = \rho$. Fig. 1 shows the change in deflections of the outer load-bearing shell ($w^* = G_2 w/p_0$) along the length at $c_{01} = 0.05$. Here, curves 1 correspond to the solution for rigid contact between the load-bearing layers and the transversely isotropic filler, curves 2 have no inner load-bearing layer, and curves 3 have a completely rigid inner layer. The dotted curves correspond to the isotropic filler.

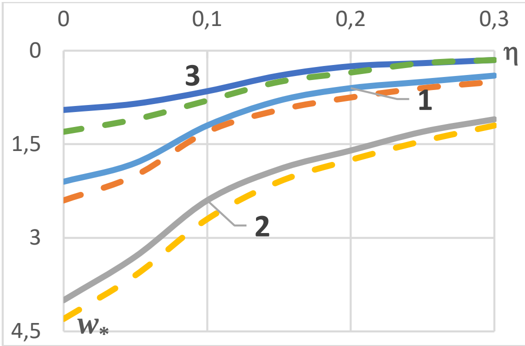

Figure 1. Distribution of shell deflections along the length

Figure 2 shows similar graphs for dimensionless ($\sigma^* = -a_2\sigma_{rr}/p_0$) contact stress on the contact surface of the outer layer and filler. Fig. 2 shows that with distance from the point of application of the concentrated ring load, the contact stresses change sign, which indicates the possibility of separation of the loaded shell from the filler in the case of sliding contact. In addition, Figures 1 and 2 show that changing the conditions on the inner surface of the filler significantly affects the deflections of the loaded bearing layer and has a much smaller effect on the contact stresses on the outer surface of the filler. The anisotropy of the filler material for the given parameter values does not change the qualitative picture, but it does change the quantitative characteristics of the stress-strain state of such a mechanical system composed of three elements.

Figure 3 shows the distribution of dimensionless radial displacements, and Figure 4 shows the distribution of dimensionless radial stresses across the thickness of the filler for different cross-sections $\eta = const$ during the movement of the ring load. The meaning of indices 1–3 is the same as in Figures 1 and 2.

It can be seen that with distance from the point of application of the load, the distribution pattern changes significantly, especially for stresses.

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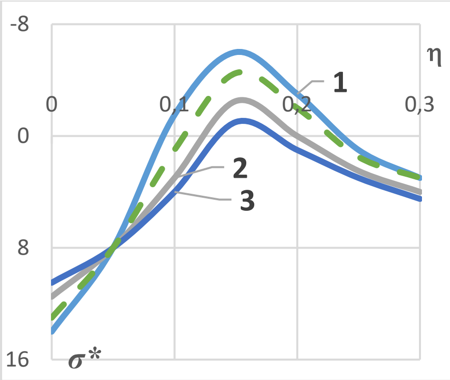
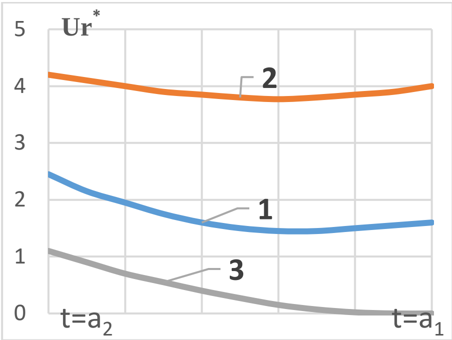
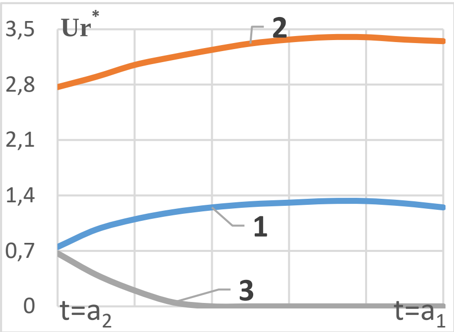


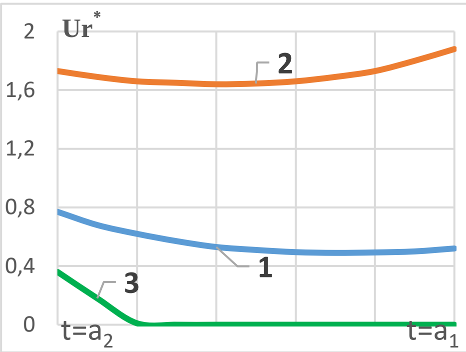
Figure 2. Contact stresses between the outer load-bearing layer and the aggregate



a η = 0

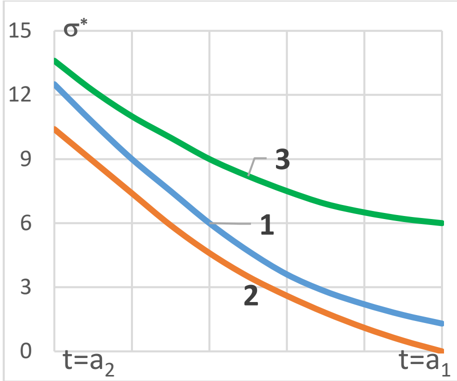


b η = 0.1

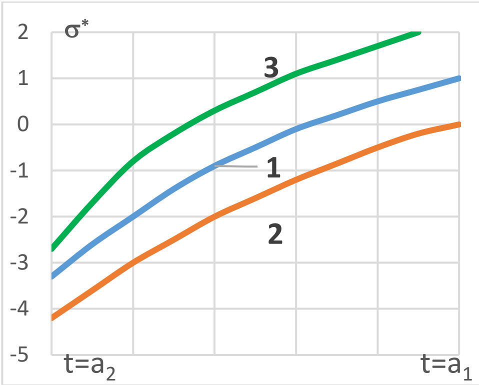


c) *η* = 0.2

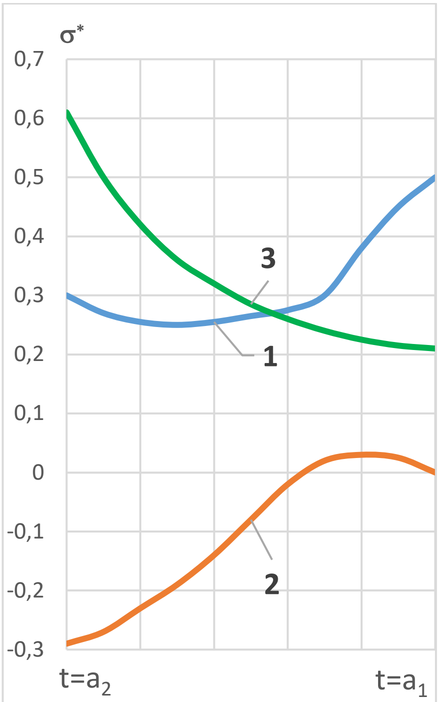
Figure 3. Radial displacements in different cross-sections of the filler



a η = 0



b η = 0.1



c η = 0.2

Figure 4. Change in normal stresses across the thickness of the filler filler

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In conclusion, we note that the approach proposed in this work to extend the authors' approach describing the dynamics of three-layer shells to the case of a filler made of transversely isotropic material opens up possibilities for its use for other types of anisotropic materials. At the same time, the main difficulties on this path may arise when integrating dynamic equations for such media.

Conclusions

1. A semi-accurate mathematical model is proposed to describe the dynamics of a three-layer cylindrical shell with a transversely isotropic filler, where the motion of the supporting layers is described by the equations of motion of such shells, which are based on the approaches of Timoshenko or Kirchhoff-Lyau, and for the filler, the exact equations of the elasticity theory of an anisotropic body are used, with the contact conditions at the boundaries of each of the three layers being satisfied.

2. To solve the stationary dynamic problem of motion along the surface of an infinitely long three-layer cylindrical shell, an algorithm has been developed that consists of using Galilean transformation and applying, in a moving coordinate system that moves together with the load, complex integral Fourier transform. In this case, potential functions are introduced in a special way using previously unknown coefficients in the image space for integrating the equations of motion of a transversely isotropic material.

3. In general, the results after applying the inverse Fourier transform are obtained in the form of improper integrals with complex subintegral functions, therefore, for their approximate calculation with a given accuracy, a program has been developed based on the method proposed by Filon for constructing quadrature formulas for the case of strongly oscillating functions.

4. Various cases of boundary conditions on the inner surface of the filler are considered as special cases of the general results, in particular, the case of a shell with a liquid filler and a shell with a filler and a rigid core inside it.

5. For all the problems considered, numerical results were obtained using a computer and corresponding graphs were constructed, showing the influence of anisotropy and boundary conditions on the stress-strain state of a cylindrical shell composed of three layers.

6. The results of the work can be used for comparison when constructing approximate models of the dynamics of three-layer cylindrical shells with both isotropic and anisotropic filler material.

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ДІЯ РУХОМОГО НАВАНТАЖЕННЯ НА ТРЬОХШАРОВУ ЦИЛІНДРИЧНУ ОБОЛОНКУ З ТРАНСВЕРСАЛЬНО ІЗОТРОПНИМ ЗАПОВНЮВАЧЕМ

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Мета роботи. *Розповсюдити раніше запропонований авторами підхід про застосування для задач динаміки трьохшарових циліндричних оболонок з ізотропним заповнювачем точних рівнянь теорії пружності на один із можливих випадків анізотропії матеріалу середнього шару, а саме ситуацію, коли заповнювач є трансверсально ізотропним. Отримати точні формули і на їх основі побудувати картину напружено-деформованого стану в такій складеній конструкції при русі вздовж зовнішньої поверхні зі сталою швидкістю нормального (радіального) навантаження.*

Методи дослідження. *Побудована математична модель динаміки трьохшарової циліндричної оболонки, коли рух несучих шарів описується рівняннями теорії тонких оболонок, а для трансверсально ізотропного заповнювача використовуються динамічні рівняння теорії пружності анізотропного середовища у загальному вигляді. При розгляді задачі у стаціонарній постановці застосовується перетворення Галілея, після чого в рухомій системі координат до усіх шуканих і заданих величин застосовується інтегральне перетворення Фур'є у комплексній формі. Для обчислення невласних інтегралів Фур'є розроблено квадратурні формули, які основані на методі Файлона для інтегрування швидко осцилюючих функцій, що дозволило ефективно отримувати чисельні результати із наперед заданою точністю.*

Отримані результати. *На основі побудованої моделі розглянута задача про рухоме навантаження, яке викликає стаціонарний напружено-деформований стан шаруватої циліндричної оболонки при різних умовах на поверхнях стику заповнювача і несучих шарів. При цьому контакт розглядається як жорсткий, так і ковзний, але виключається відставання оболонок від заповнювача. Складнощі, які виникають при розв'язанні рівнянь руху трансверсально ізотропного заповнювача здолані шляхом введення спеціальним способом з використанням невизначених коефіцієнтів потенціальних функцій. Для усіх можливих варіантів граничних умов результати отримані у вигляді неособливих невласних інтегралів, які обчислені за спеціальними квадратурними формулами. Показані картини розподілу переміщень і напружень як за довжиною так і за товщиною заповнювача, проведено порівняння з результатами для відповідного ізотропного заповнювача і проведено механічний аналіз результатів.*

Наукова новизна*. Вперше в такій постановці, коли поведінка заповнювача описується точними рівняннями динаміки пружного анізотропного тіла, отримано розв'язання стаціонарної динамічної задачі для трьохшарової циліндричної оболонки. Проведено порівняння з результатами раніше отриманими для випадку ізотропного заповнювача. Застосовано спеціальний прийом для введення потенціальних функцій для знаходження переміщень*

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і напружень в динамічних рівняннях для трансверсально ізотропних матеріалів. Розглянуті важливі частинні граничні умови на границях контактів шарів.

Практична цінність. *Отримані при такому підході результати можуть бути використані в якості еталонних при побудові спрощених моделей динамічної поведінки трьохшарових циліндричних оболонок, зокрема таких, які враховують анізотропію заповнювача. До таких можна для прикладу віднести так звані ребристі звукоізолюючі матеріали.*

Ключові слова*: Шарові оболонки, трансверсально ізотропне середовище, інтегральне перетворення, потенціальні функції, граничні умови, переміщення, напруження.*

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