UDC 621.316.13

Volodymyr Shevchenko	Candidate of Technical Sciences, Head of the Department of Theoretical and Applied Mechanics, National University Zaporizhzhia Polytechnic, Zaporizhzhia, Ukraine, *e-mail: shevawk@gmail.com,* ORCID: 0000-0001-9037-6367
Andrii Skrebtsov	Candidate of Technical Sciences, Associate Professor of the Department of Theoretical and Applied Mechanics, National University Zaporizhzhia Polytechnic, Zaporizhzhia, Ukraine, *e-mail: nic_tz@ukr.net,* ORCID: 0000-0002-4669-9625
Lina Zlatkin-Blank	Master of Mathematics, School "Asif" Misgav, Nazareth, Ellit, Israel, *e-mail: zlatkinl@gmail.com,* ORCID: 0009-0007-6065-2106
Svetlana Kruzhnova	Senior lecturer of the Department of Theoretical and Applied Mechanics, National University Zaporizhzhia Polytechnic, Zaporizhzhia, Ukraine, *e-mail: krulana2017@gmail.com*, ORCID: 0000-0002-7554-0322.
Olga Omelchenko	Senior lecturer of the Department of Theoretical and Applied Mechanics, National University Zaporizhzhia Polytechnic, Zaporizhzhia, Ukraine, *e-male: omelchenko15@ukr.net,* ORCID: 0000-0001-8925-4178
Natalia Shaleva	Assistant lecturer of the Department of Theoretical and Applied Mechanics , National University Zaporizhzhia Polytechnic, Zaporizhzhia, Ukraine, *e-mail: shaleva1711@ukr.net,* ORCID: 0000-0002-6571-0359

TAKING FRICTION INTO ACCOUNT IN KINETOSTATIC ANALYSIS OF MECHANISMS

Purpose. *Study the dynamics and characteristics of friction using the example of translational kinematic pairs.*

Research methods. *The theoretical aspects of friction research in translational kinematic pairs are examined in detail. The introduction of friction forces into the equations of kinematics leads to an increase in the number of unknown reaction components in kinematic pairs, while the number of equations remains unchanged. The force calculation of mechanisms taking into account friction comes down to a joint solution of kinematics equations containing friction forces as additional unknowns and relations obtained while considering the corresponding models of kinematic pairs of friction.*

Results. *As a result, analytical dependencies were obtained for determining the speed, acceleration and reaction from the magnitude of the slider displacement, and changes in the power parameters of the piston pump due to wear of the parts of the reciprocating slider-guide pair were analyzed.*

Scientific novelty. *The current level of technological progress requires constant improvement of product quality and productivity to make it competitive. This leads to increased requirements for the performance characteristics of moving joints in mechanisms and machines operating under extreme conditions of friction and wear. Friction forces arise in the kinematic pairs of mechanisms, and in many cases these forces significantly affect the movement of the mechanism links and must be taken into account in force calculations. The energy costs associated with overcoming harmful resistance forces are irreversible, and the reduction of irreversible energy costs is achieved by limiting friction forces.*

Practical value. *The results of the research showed that the wear of the parts of the translational kinematic pair of the slider and the pump guides leads to an insignificant change in both the speed, and acceleration of the slider, at the same time, the maximum reaction in this kinematic pair changes more significantly.*

Key words: *kinetostatics, kinematic pair, friction, wear, resistance reactions, equilibrium equations.*

Introduction

Every new step in the development of machines, mechanisms and devices is associated with a deep study of the phenomena occurring on the contact surfaces of parts, taking into account their strength, material properties and the peculiarities of destruction processes. In the context of combating wear, the development of a general theory of material wear becomes particularly relevant, which allows not only to predict the resource of parts, but also to increase the reliability and efficiency of modern technical systems. [1].

The general problem of reliability, accuracy and durability of machines, mechanisms and devices is mainly related to issues of friction, lubrication, wear of surfaces of parts and working bodies that interact with each other in complex conditions.

It has been established that today approximately one-third to one-half of the world's energy resources are spent in one form or another on friction. In translational kinematic pairs, friction has a complex nature, which changes dynamically depending on the speed, load, surface condition and lubrication. This directly affects the efficiency, durability and reliability of aircraft pumps, engines, hydraulic

DOI 10.15588/1607-6885-2025-3-8

drives. Therefore, the importance of the problem of friction and wear of machine parts in today's highly mechanised world cannot be overestimated [1]. Since the wear of moving joints under the influence of friction forces leads to premature failure of machines and unjustifiably high repair costs, special attention is paid to preventing wear in machines and mechanisms.

Most problems are solved using analytical calculation methods. Analytical methods can be divided into: simplified (classical (Coulomb friction), energy method) – for educational and approximate problems and advanced (Dahl model, LuGre friction model) – for accurate modelling in real mechanisms.

However, the obtained results often differ from the actual loads in the mechanisms. In addition, these calculations do not take into account the influence of aggressive factors of corrosive and abrasive environments, in which the equipment often operates [2].

It should be noted that the problem of determining friction coefficients and corresponding losses of parts and mechanisms has not been fully resolved. Determination of friction coefficients and corresponding energy losses in parts and mechanisms is one of the key and not yet fully solved problems of modern tribology and the theory of mechanisms. The problem of determining friction coefficients and energy losses in mechanisms remains open, since: the friction coefficient is not a constant value, its value depends on a complex set of factors, calculation models require experimental confirmation. Today, hybrid methods are used (a combination of theoretical models, computer simulations and experimental measurement.

Therefore, considering the abovementioned, the aim is to study tools and methods for analysing friction processes in kinematic pairs, which will enable the development of proposals for structural improvements, taking into account energy consumption in the system.

Analysis of research and publications

It is known that the forces counteracting displacement in any system are represented by useful and harmful resistance forces. The ratio of these forces determines the efficiency and durability of the system. The designer's task is to minimize harmful resistance (through lubrication, selection of materials, balancing of mechanisms). Useful or technological resistance is determined by the characteristics of the mechanism itself and, for given conditions, in most cases cannot be varied [3, 4]. In contrast, it is possible to limit harmful resistance within certain limits.

Equipment efficiency, with all other things being equal, is linked to a bunch of kinematic parameters in the chain from the engine to the working body, as well as the ability to create parallel or branched flows for specific purposes [3]. In the solution of problems of synthesis of technological machines, it is precisely the kinematic part that plays a decisive role. This is especially true for machines with periodic (cyclical) action, since the set of transient processes associated with their start and stop is supplemented by transient processes caused by reciprocating movements of links. In addition to these, in many cases, kinematic or dynamic disturbances are added, which are a consequence of the structural features of the machines themselves.

Today, the development of automatic machine theory is mainly driven by the need to improve control system design methods that ensure the coordination of executive bodies. This is because the energy costs associated with overcoming harmful resistance forces (friction forces) are irreversible and are accompanied by the conversion of mechanical energy of motion into thermal energy [5].

Irreversible energy losses are reduced by limiting friction forces, which are mainly determined by the force interaction between moving and stationary parts and the values of friction coefficients [4]. The former depend on the condition and materials of contact surfaces, friction modes, relative sliding speeds, preliminary contact time, etc.

The necessity to study the wear resistance of machine parts is caused by significant economic costs of repairs and upgrades. High costs for repair and replacement of parts: in industry, up to 70 % of machine downtime is due to friction and wear. Reduced efficiency and productivity: wear increases energy losses due to friction, increases fuel and electricity consumption. Extending the life of parts directly reduces maintenance costs.

According to tribologists, in highly developed countries, losses in mechanical engineering associated with friction and wear reach about 8 % of national income. These losses are formed due to increased costs for fuel and energy (due to increased friction forces); repairs and replacement of parts that wear out quickly; a decrease in the efficiency of machines and mechanisms; forced downtime of production. Thus, even a 1% reduction in friction on the scale of the national economy can provide a colossal economic effect, namely billions of savings on repairs and energy resources; an increase in the resource of machines and mechanisms; a decrease in the cost of production; an increase in the competitiveness of the industry [6].

With the development of scientific and technological progress, the need for complex calculations of resistance forces in loaded, automatic and particularly precise friction units has become relevant, and the need to ensure their antifriction properties under operating conditions has also arisen. Today, many industries require the creation of special devices and braking systems, where the work is directly based on the laws of friction. For example, aviation and transport – wheel and disc brakes, aircraft landing brake systems; mechanical engineering – brake clutches, clamping devices, friction gears; energy – turbine and generator braking systems; robotics and mechatronics – precision control systems, where the use of friction provides positioning and stabilization.

Thus, the study of the interaction of surfaces in relative motion, as well as the consequences associated with this phenomenon, and the determination of the possibility of influencing friction coefficients in the direction of their reduction, is a relevant scientific task today, which has significant theoretical and practical significance. The study of friction between moving surfaces and methods for its re-

DOI 10.15588/1607-6885-2025-3-8

duction is an urgent task of modern science and technology. This research is important both from a theoretical point of view - for a better understanding of the processes of interaction of materials, and from a practical point of view - for increasing the reliability, energy efficiency and durability of machines and mechanisms.

Nowadays, the effectiveness of friction research in kinematic pairs, which affects moving joint systems during mechanical testing, has been demonstrated in several scientific studies [7–9]. A structural-energy theory of friction and wear in machines has also been developed, which is widely known throughout the world and became the basis for modern methods of calculating the service life of machine parts, selecting materials and lubricants. It is actively used in aviation, mechanical engineering, transport and energy, where the problem of wear is particularly relevant [10].

At the same time, despite the significant scientific heritage that exists today, questions remain regarding the justification of optimal design solutions, as well as the correct choice of materials and the determination of rational technological methods for manufacturing and strengthening machine parts.

Purpose

The aim of the study is to identify the patterns of dynamics and characteristics of friction processes in translational kinematic pairs, determine their impact on the force and kinematic parameters of mechanisms, and develop approaches to account for friction (including the influence of wear and changes in friction coefficients) in kineto-static analysis to improve the accuracy of calculations, reliability and durability of machines and devices.

Therefore, taking into account the abovementioned, the purpose of this article is to study the dynamics and characteristics of friction using the example of translational kinematic pairs.

Material and research methods

Friction is the resistance that arises when one body moves relative to another. The surfaces that come into contact with each other are called friction surfaces. There are two main types of friction: sliding friction and rolling friction [2].

In lower kinematic pairs, sliding friction prevails, while in higher pairs, rolling friction or a combination of rolling and sliding friction may occur.

First, let us consider in detail the theoretical aspects of friction research in translational kinematic pairs.
Therefore, friction occurs when the slider slides along a horizontal plane.

Fig. 1 shows a moving kinematic pair consisting of a horizontal guide 2 and a slider 1.

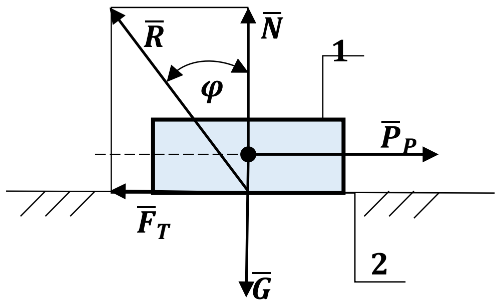

Figure 1. Kinematic pair with translational motion

Let the following forces act on slider 1: $\bar{\boldsymbol{P}}_{\boldsymbol{P}}$ – translational force, $\bar{\boldsymbol{G}}$ – weight of the load or load acting on the slider, $\bar{\boldsymbol{N}}$ – normal reaction, $\bar{\boldsymbol{F}}_{\boldsymbol{T_0}}$ – friction force (tangential reaction) at rest. When the slider moves, instead of the friction force at rest, the friction force of motion $\bar{\boldsymbol{F}}_{\boldsymbol{T}}$ acts, where $\boldsymbol{N} = -\boldsymbol{G}$ and the total reaction

$$\boldsymbol{R} = \boldsymbol{F_T} + \boldsymbol{N}.$$

The angle $\boldsymbol{\varphi}$ of deviation of the total reaction from the normal in the direction opposite to the movement of the slider is called the friction angle. Therefore:

$$\boldsymbol{F_{T_0}} = \boldsymbol{N} \cdot \mathbf{tg}\boldsymbol{\varphi_0}; \quad \boldsymbol{F_{T_0}} = \boldsymbol{f_0} \cdot \boldsymbol{G}; \ \textbf{where} \ \ \boldsymbol{f_0} = \mathbf{tg}\boldsymbol{\varphi_0}.$$

Considering that

$$\boldsymbol{F_T} = \boldsymbol{N} \cdot \mathbf{tg}\boldsymbol{\varphi} = \boldsymbol{G} \cdot \mathbf{tg}\boldsymbol{\varphi}; \ \ \boldsymbol{F_T} = \boldsymbol{f} \cdot \boldsymbol{G},$$

we will have

$$\boldsymbol{f} = \mathbf{tg}\boldsymbol{\varphi}.$$

Therefore, the coefficient of friction is equal to the tangent of the angle of friction. If the direction of the slider's movement is changed, the total reaction will deviate accordingly. The geometric locus of total reactions is the lateral surface of a straight cone (Fig. 2).

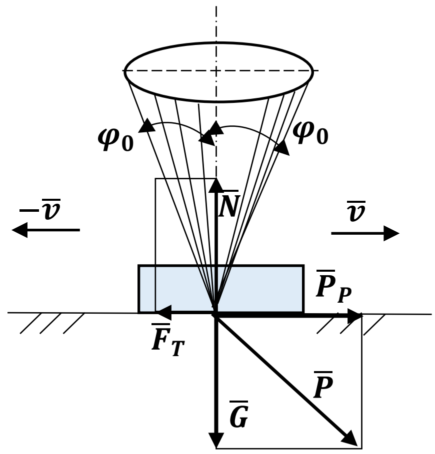

Figure 2. Friction cone

DOI 10.15588/1607-6885-2025-3-8

The friction cone has the following features:

- if the resultant $\boldsymbol{P}$ of the driving force $\boldsymbol{P_P}$ and the gravitational force $\boldsymbol{G}$ passes inside the friction cone, then $\boldsymbol{P_P} < \boldsymbol{F}$. Therefore, the body will not move from its place;
- if the resultant $\boldsymbol{P}$ of the driving force $\boldsymbol{P_P}$ and the force of gravity $\boldsymbol{G}$ passes outside the friction cone, then $\boldsymbol{P_P} > \boldsymbol{F}$ and the body will move from its place.

Obviously, when starting from rest $\boldsymbol{P_P} > \boldsymbol{f_0} \cdot \boldsymbol{G}$; under equilibrium conditions, $\boldsymbol{P_P} < \boldsymbol{f_0} \cdot \boldsymbol{G}$; in the case of uniform motion, $\boldsymbol{P_P} = \boldsymbol{f} \cdot \boldsymbol{G}$; in the case of accelerated motion, $\boldsymbol{P_P} > \boldsymbol{f} \cdot \boldsymbol{G}$.

Fig.3 shows the dependence of the friction force $\boldsymbol{F_T}$ on the driving force $\boldsymbol{P_P}$, $(\boldsymbol{F_T} = \boldsymbol{f}(\boldsymbol{P_P}))$.

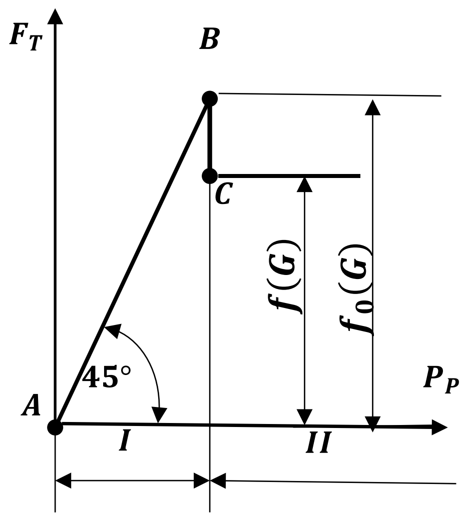

Figure 3. The graph of the dependence of $\boldsymbol{F_T}$ on $\boldsymbol{P_P}$.

Point A corresponds to the state of rest $\boldsymbol{P_P} = \boldsymbol{0}$ and $\boldsymbol{P} = \boldsymbol{0}$; section I corresponds to the state of equilibrium $\boldsymbol{P_P} \neq \boldsymbol{0}$, but $\boldsymbol{v}_{\text{к}} = \boldsymbol{0}$; section II corresponds to the state of motion $\boldsymbol{P_P} \neq \boldsymbol{0}$ and $\boldsymbol{v}_{\text{к}} \neq \boldsymbol{0}$; the limiting friction force $\boldsymbol{F_{T_0}} = \boldsymbol{f_0} \cdot \boldsymbol{G}$. In this case, friction force is understood as the force of adhesion. The values of the coefficients of sliding friction are given in Table 1. It should be remembered that these values are approximate and may vary depending on surface quality, temperature and other factors.

Table 1 – Friction coefficients in various kinematic pairs

Material of the pair	f_0		f	
	dry surfaces	lubricated	dry surfaces	lubricated
Steel on steel	0.15	0.11	0.13	0.09
Steel on bronze	0.11	0.10	0.10	0.09
Copper on steel	0.53	0.36	0.36	0.18
Brass on steel	0.51	0.19	0.35	0.16

In addition to the design, technological and operational factors discussed above, the shape and location of the elements of the kinematic pair also affect the friction coefficient [10–12]. For different types of pairs, reduced friction coefficients are determined [9].

Now let's take a closer look at how the characteristics of the friction process are determined in practice.

For example, piston pumps are widely used in oil and gas production, as well as in oil refining, since other types of pumps are unsuitable due to the intensive wear of hydraulic parts. There are studies that examined wear and considered various methods of increasing the durability of parts of the hydraulic part of piston pumps, but without taking into account the wear of parts of the mechanical drive part [13, 14]. There are also works [15] in which data for the kinematic and force calculation of the crank-slider mechanism of the pump was obtained. However, the authors did not take into account the wear of the reciprocating pair ‘crosshead (slider) – guides’.

Let us analyse the changes in the kinematic and force parameters of a piston pump due to wear of the reciprocating pair of parts: the crosshead (slider) and the pump frame guides.

In the drive of piston pumps for converting rotary motion into reciprocating motion, a crank-slider mechanism with one degree of freedom is used (Fig. 4).

To study the effect of slider displacement due to wear of the kinematic pair parts of the crosshead (slider) – guide rails of a horizontal type piston pump, two-cylinder, double-acting (cranks are positioned at an angle of 90 ° in the direction of rotation) on its kinematic and power parameters, equations of closed vector contours *OABCO* and $\boldsymbol{OA_1B_1CO}$ were compiled. To simplify, we assume that the displacement of the slider does not lead to a skew of the rod axis, and we give formulas only for one contour $\boldsymbol{OABCO}$. Let us write down the vector equation:

$$\boldsymbol{\ell_1} + \boldsymbol{\ell_2} = \bar{\boldsymbol{e}} + \bar{\boldsymbol{x}}_{\boldsymbol{B}}.$$

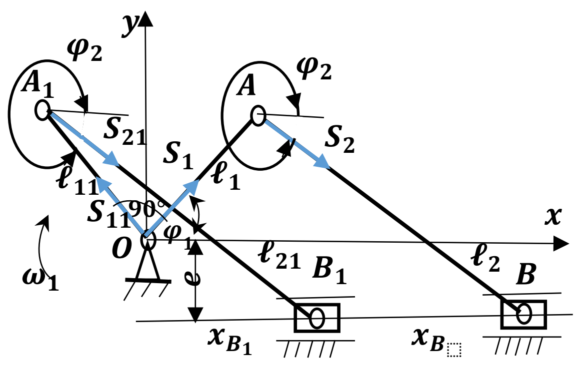

Figure 4. Kinematic scheme of the drilling pump mechanism

After performing the appropriate transformations, we obtain the following ratios:

DOI 10.15588/1607-6885-2025-3-8

$$x_A = \ell_1 \cos\varphi_1;$$

$$y_{S_1} = \ell_{S_1} \sin\varphi_1;$$

$$x_{B_1} = \ell_1 \cos\varphi_1 + \sqrt{\ell_2^2 - (\ell_1 \sin\varphi_1 + e)^2};$$

$$y_A = \ell_1 \sin\varphi_1; \;\; x_{S_2} = \ell_1 \cos\varphi_1 + \ell_{S_2} \cos\varphi_2; \quad y_B = e;$$

$$x_{S_1} = \ell_{S_1} \cos\varphi_1; \;\; y_{S_1} = \ell_{S_1} \sin\varphi_1 + \ell_{S_2} \sin\varphi_2.$$

Here e - s the displacement of the slider due to wear of the crosshead pads and pump frame guides.

As a result of differentiation of the aforementioned equations by the generalised coordinate φ_1 we obtain the dependencies of the change in projections of the analogues of the velocities of point A, centres of mass S_1 of crank OA, S_2 of connecting rod AB and slider B on the angle of rotation of the crank and the displacement e of slider B respectively:

$$x'_A = -\ell_1 \sin\varphi_1; \;\; y'_{S_1} = \ell_{S_1} \cos\varphi_1;$$

$$x'_B = \ell_1 \mathrm{tg}\varphi_2 \cdot \cos\varphi_1 - \ell_1 \sin\varphi_1;$$

$$y'_A = \ell_1 \cos\varphi_1;$$

$$x'_{S_2} = -\ell_1 \sin\varphi_1 - \ell_{S_2} \sin\varphi_2 \cdot \varphi_2{}'; \quad y_B{}' = 0;$$

$$x'_{S_1} = -\ell_{S_1} \sin\varphi_1; \;\; y'_{S_1} = \ell_1 \cos\varphi_1 + \ell_{S_2} \cos\varphi_2 . \varphi'_2.$$

where $x'_A; y'_A; x'_{S_1}; y'_{S_1}; x'_{S_2}; y'_{S_2}; x'_B; y'_B$ – projections of the analogues of the velocities of points A and B, centres of mass S_1 and S_2 from the crank rotation angle φ_1 and the displacement value e of slider B respectively. The rotation angle of connecting rod AB:

$$\varphi_2 = \arcsin\left(-\frac{\ell_1 \sin\varphi_1 + e}{\ell_2}\right).$$

Having differentiated the above dependencies of projection changes in a similar way, we obtain the dependencies of projection changes of analogues of accelerations of point A, centres of mass S_1 of crank OA, S_2 of the connecting rod AB and slider B from the crank rotation angle φ_1 and the displacement value e of slider B.

We use the obtained results of the kinematic calculation to perform a force analysis of the pump mechanism and study the effect of the slider displacement on the reactions in the translational kinematic pair of the crosshead and the pump frame guides. To do this, we analyse the structural group of the connecting rod and slider using the principle of kinematics (Fig. 5).

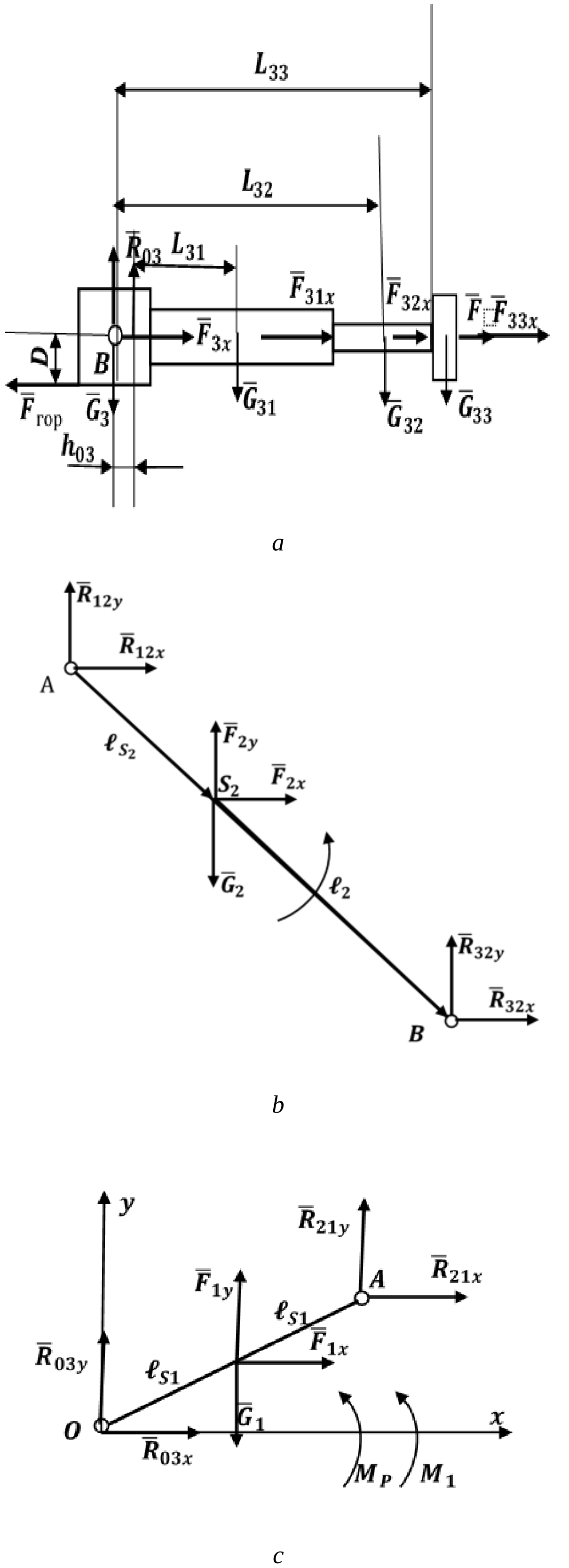

Figure 5. Force analysis of the pump mechanism:
a – slider *B*; *b* – connecting rod *AB*; *c* – crank *OA*

DOI 10.15588/1607-6885-2025-3-8

The kinematic equations for each link of the pump mechanism are as follows:
for connecting rod $\boldsymbol{AB}$:

$$\begin{cases} \boldsymbol{R_{12x} + F_{2x} + R_{32x} = 0} \\ \boldsymbol{R_{12y} + F_{2y} + R_{32y} - G_2 = 0} \\ \boldsymbol{M_B R_{12} + M_B F_2 + M_B G_2 + M_2 = 0}; \end{cases}$$

for slider *B*:

$$\begin{cases} \boldsymbol{R_{23x} + F_{3x} + R_{31x} + F_{32x} + F_{33x} +} \\ \boldsymbol{F - \mu_1(v_B) R_{03} = 0} \\ \boldsymbol{R_{23y} + R_{01} - G_3 + G_{31} - G_{32} - G_{33} = 0} \\ \boldsymbol{R_{03} \cdot h_{03} + \mu_1(v_B) R_{03}^{0.5} D - G_{31}\ell_{31} -} \\ \boldsymbol{G_{32}\ell_{32} - G_{33}\ell_{33} = 0}; \end{cases}$$

for crank *OA*:

$$\begin{cases} \boldsymbol{R_{01x} + R'_{21x} + R_{21x} + F_{1x} + F_{11x} = 0} \\ \boldsymbol{R_{01y} + R_{21y} + R'_{21y} - G_1 + G_{11} + F_{1y}} \\ \boldsymbol{+F_{11y} = 0} \\ \boldsymbol{M_O R_{21} + M_O R'_{21} + M_O F_1 + M_O F_{11} +} \\ \boldsymbol{M_O G_1 + M_O G_{11} + M_1 + M_P = 0}; \end{cases}$$

where $\boldsymbol{R_{ijx}}$, $\boldsymbol{R_{ijy}}$ – are projections of the force (reaction) acting on the *i-th link;*

$\boldsymbol{F_{ijy} = -a_{ixy} m_i}$ – is the projection of the inertia force of the i-th link;

$\boldsymbol{F}$ – is the resistance force applied to the slider, taking into account the friction forces in the pairs: seal – rod and piston – sleeve;

$\boldsymbol{G_i}$ – is the gravitational force of the i-th link;

$\boldsymbol{D}$ – is the diameter of the slider (‘crosshead’);

$\boldsymbol{M_i = -E_i J_i}$ – is the moment of inertia forces of the i-th link;

$\boldsymbol{\mu_1}$ – friction coefficient in the translational kinematic pair of the crosshead and the pump guide rails.

Research results and their discussion

The solutions of the systems of equations are used to determine the reactions in the kinematic pairs of the pump mechanism for different values of the crank rotation angle $\boldsymbol{\varphi_1}$ and different displacements *e* of the slider *B*. Based on the obtained data, it is possible to construct graphical dependencies $\boldsymbol{R_{03max} = f(S_B)}$ for the maximum – 25 MPa (piston diameter $\boldsymbol{D_n}$ = 130 mm) and minimum – 10 MPa (piston diameter $\boldsymbol{D_n}$ = 200 mm) values of pump pressure at $\boldsymbol{e = 0}$, for direct (piston movement into the rodless chamber of the pump) and reverse (piston movement into the rod chamber of the pump) pump strokes. The stroke of the pump piston is = 0,4 m.

For the range $\boldsymbol{e = \pm 5}$ mm, the dependence of the maximum reaction in the reciprocating pair for both forward and reverse pump strokes is linear. For the forward stroke of the pump, the maximum reaction increases with an increase in the displacement of the slider (direct relationship), and for the reverse stroke, the maximum reaction decreases with an increase in the displacement of the slider (reverse relationship). That is, for forward stroke, an increase in the slider displacement (vertically down) leads to an increase in the maximum reaction in the reciprocating pair, which in turn leads to increased wear of the slider linings and pump guide rails.

The results of the research showed that the wear of the parts of the translational kinematic pair of the crosshead (slider) and the pump guides leads to an insignificant change in both the speed (0.07 %), and acceleration (0.0014 %) of the slider *B*. At the same time, the maximum reaction in this kinematic pair changes more significantly. During the forward stroke of the piston, it increases by 2.5 %, and during the reverse stroke of the piston, it decreases by 2.4 %.

Conclusions

The theoretical and practical aspects of friction research in translational kinematic pairs and models for determining velocity, acceleration, and reaction from slider displacement using the example of a piston pump described in this article make it possible to more accurately determine their numerical values during the wear of certain parts of machines and mechanisms, in particular the slider and the pump guide rails. These results should be used to select modes during testing for wear of materials for parts of translational kinematic pairs.

External friction in braking systems and special devices acts not only as a resistance factor, but as a controlled technological element that requires accurate mathematical description, prediction, and effective use.

Thus, in conclusion, it can be noted that the theoretical analysis of the friction process in translational kinematic pairs conducted during the study makes it possible to accurately assess the level of losses from wear and tear of parts, taking into account the values of the design parameters of mechanisms, and can be used to predict the effective performance of the designed technological equipment.

References

1. Zamri Yusoff, Shamsul Baharin Jamaludin (2011). Tribology and Development of Wear Theory: Review and Discussion. International Journal of Current Research and Review IJCRR – 3(2), 13–26.

2. Shaonian Li, Hao Liu, Yi Li, Shangling Bao, Pan Yang (2021). Tribological Behavior of Friction Pairs in a High-Pressure Vane Pump under Variable Working Conditions. Tribology Online, Volume 16 Issue 3 Pages 159-169. https://doi.org/10.2474/trol.16.159

3. Kinytsky Ya.T. (2002). Theory of mechanisms and machines. Textbook. Naukova Dumka, 660.

4. SkrebtsovA., Omelchenko O., Kruzhnova S., Shaleva N. (2023). Lecture notes on “Theory of mechanisms and machines”. Zaporizhzhia: National University “Zaporizhzhia Polytechnic”, 266.

5. Yuan, W., Dong, G., Chin, K. S., & Hua, M. (2016). Tribological assessment of sliding pairs under damped harmonic excitation loading based on on-line monitoring

DOI 10.15588/1607-6885-2025-3-8

methods. Tribology International, 96, 225–236. https://doi.org/10.1016/j.triboint.2015.12.044

6. Feshchenko A. V., Lyashenko S. V., Topolsky O. I. (2022). The influence of the friction coefficient on the efficiency of machine operation. NTU “KhPI”, 15–22.

7. V. O. Kharzhevsky, M. V. Marchenko, V. O. Korzhenko (2022). Force calculation of lever mechanisms of class II taking into account friction forces in kinematic pairs. Modern achievements in science and education: collection of proceedings of the XVII International Scientific Conference, Netanya (Israel). Khmelnytskyi: KhNU, 89–93.

8. Kharlamov Yu. O., Polonsky L. G., Balytska N. O., Melnyk O. L., Nochvai V. M. (2021). Classification of machine parts connections according to wear conditions. Bulletin of the Khmelnytsky National, 79–86.

9. Mazhara V. A. (2025). Research on the influence of friction in the hinges of the gripping device on the clamping force. LVI scientific conference of teachers, postgraduate students and university employees (TSNTU), 115–117.

10. W. Qian, S. Zhang, and Y. Liu (2022). Effect of a kinematic pair containing clearance and friction on dynamics, 12, 1, 117.

11. Kelemenová, T., Dovica, M., Božek, P., Koláriková, I., Benedik, O., Virgala, I., Prada, E., Miková, Ľ., Kot, T., & Kelemen, M. (2020). Specific Problems in Measurement of Coefficient of Friction Using Variable Incidence Tribometer. Symmetry, 12(8), 1235. https://doi.org/10.3390/sym12081235

12. T. Kelemenová, Z. Tkáč, and M. Varga (2020). Specific problems in the measurement of coefficient of friction in sliding pairs. Acta Mech. Slovaca, 24, 3, 30–35.

13. Zhang, X., Li, B., Wang, X. (2022). Research on wear and friction characteristics in friction pairs of a high-pressure hydraulic pump. Tribology International, 167.

14. Bian, B. (2024). Lubrication, friction, and wear characteristics of textured slipper pairs in axial piston pumps. Lubricants, 12(11), 370. https://doi.org/10.3390/lubricants12110370

15. He, Y., Tan, M., & Lu, H. (2022). Kinetostatic modeling and analysis of compliant mechanisms. Meccanica, 57, 991–1012.

Received 28.08.2025

УРАХУВАННЯ ТЕРТЯ ПРИ КІНЕТОСТАТИЧНОМУ АНАЛІЗІ МЕХАНІЗМІВ

Володимир Шевченко	канд. техн. наук, доцент, завідувач кафедри теоретичної та прикладної механіки Національного університету «Запорізька політехніка», м. Запоріжжя, Україна, *e-mail: shevawk@gmail.com,* ORCID: 0000-0001-9037-6367
Андрій Скребцов	канд. техн. наук, доцент кафедри теоретичної та прикладної механіки Національного університету «Запорізька політехніка», м. Запоріжжя, Україна, *e-mail: nic_tz@ukr.net,* ORCID: 0000-0002-4669-9625
Ліна Златкін-Бланк	викладач математики школи «Асіф» Місгав, Назарет, Елліт, Ізраїль, *e-mail: zlatkinl@gmail.com,* ORCID: 0009-0007-6065-2106
Світлана Кружнова	старший викладач кафедри теоретичної та прикладної механіки Національного університету «Запорізька політехніка», м. Запоріжжя, Україна, *e-mail: krulana2017@gmail.com,* ORCID: 0000-0002-7554-0322
Ольга Омельченко	старший викладач кафедри теоретичної та прикладної механіки Національного університету «Запорізька політехніка», м. Запоріжжя, Україна, *e-mail: omelchenko15@ukr.net,* ORCID: 0000-0001-8925-4178
Наталія Шалева	асистент кафедри теоретичної та прикладної механіки Національного університету «Запорізька політехніка», м. Запоріжжя, Україна, *e-mail: shaleva1711@ukr.net,* ORCID: 0000-0002-6571-0359

Мета роботи. *Дослідження динаміки та особливості процесів тертя на прикладі поступальних кінематичних пар.*

Методи дослідження. *Детально розглянуті теоретичні аспекти дослідження тертя в поступальних кінематичних парах. Введення сил тертя в рівняння кінетостатики призводить до збільшення числа невідомих компонент реакцій в кінематичних парах, а кількість рівнянь при цьому залишається незмінною. Силовий розрахунок механізмів з урахуванням тертя зводиться до сумісного рішення рівнянь кінетостатики, що містять сили тертя в якості додаткових невідомих, та співвідношень, отриманих при розгляданні відповідних моделей кінематичних пар із тертям.*

Отримані результати. *Отримані аналітичні залежності для визначення швидкості, пришвидшення і реа-*

DOI 10.15588/1607-6885-2025-3-8

кції від величини зміщення повзуна, проаналізовано зміни силових параметрів поршневого насоса внаслідок зношування деталей зворотньо-поступальної пари повзун – напрямні.

Наукова новизна. *Сучасний рівень технічного прогресу потребує постійного вдосконалення продукції за якістю та продуктивністю, роблячи її конкурентною. Це призводить до підвищення вимог щодо експлуатаційних характеристик рухомих з'єднань в механізмах і машинах, які працюють в екстремальних умовах тертя та зношування. В кінематичних парах механізмів виникають сили тертя і в багатьох випадках ці сили істотно впливають на рух ланок механізмів і повинні враховуватись при силових розрахунках. Енерговитрати, пов'язані з подоланням сил шкідливого опору, є незворотними, а зменшення незворотних енергетичних витрат здійснюється завдяки обмеженню сил тертя.*

Практична цінність. *Дослідження показали, що зношування деталей поступальної кінематичної пари повзун – напрямні насоса призводить до несуттєвої зміни як швидкості, так і прискорення повзуна, при цьому величина максимальної реакції в цій кінематичній парі змінюється істотніше.*

Ключові слова: *кінетостатика, кінематична пара, тертя, зношування, реакції опор, рівняння рівноваги.*

Список літератури

1. Zamri Yusoff Tribology and Development of Wear Theory: Review and Discussion / Zamri Yusoff, Shamsul Baharin Jamaludin // Conference: International Journal of Current Research and reviewт At: Nagfur, India, 2011. – Volume: Vol. 03, Iss. 02, 13–26 p.
2. Li, Y. Tribological behavior of high-pressure vane pumps under variable operating conditions / Li, Y., Guo, X., & Zhao, L. // Tribology Online Wear, 2021. – 477 p. https://doi.org/10.1016/j.wear.2021.203799
3. Кіницький, Я. Т. Теорія механізмів і машин. Підручник. – К. : Наукова думка.2002. – 660 с.
4. Конспект лекцій з "Теорії механізмів і машин" для студентів інженерних спеціальностей усіх форм навчання / Укл. А.А. Скребцов, О.С. Омельченко, С.Ю. Кружнова, Н.В. Шалева. – Запоріжжя: НУ «Запорізька політехніка». 2023. – 266 с.
5. Yuan, W. Tribological assessment of sliding pairs under damped harmonic excitation loading based on on-line monitoring methods / Wei Yuan , Guangneng Dong, K.S. Chin, Meng Hua // Tribology International Volume 96, April 2016. – P. 225–236. https://doi.org/10.1016/j.triboint.2015.12.044
6. Фещенко, А. В. Вплив коефіцієнта тертя на ефективність роботи машин / Фещенко А. В., Ляшенко С. В., Топольський О. І. // Вісник НТУ «ХПІ». – 2022. – № 2. – С. 15–22.
7. Харжевський, В. О. Силовий розрахунок важільних механізмів II класу з врахуванням сил тертя у кінематичних парах / В. О. Харжевський, М. В. Марченко, В. О. Корженко // Сучасні досягнення в науці та освіті: зб. пр. XVII Міжнар. наук. конф., 22–29 вересня 2022 р., м. Нетанія (Ізраїль). – Хмельницький : ХНУ, 2022. – С. 89–93.
8. Харламов, Ю. О. Класифікація з'єднань деталей машин за умовами зношування / Харламов Ю. О., Полонський Л. Г., Балицька Н. О., Мельник О. Л., Ночвай В. М. // Вісник Хмельницького національного університету. – 2021. – С. 79–86.
9. Мажара, В. А. Дослідження впливу тертя в шарнірах захватного пристрою на силу затиску // День науки – 2025. LVI наукова конференція викладачів, аспірантів та співробітників університету (ЦНТУ). – 2025. – С. 115–117.
10. Qian, W. Effect of a kinematic pair containing clearance and friction on dynamics / W. Qian, S. Zhang, and Y. Liu // Applied Sciences. – 2022.– Vol. 12, No. 1, Article No. 117.
11. Kelemenová, T. Specific Problems in Measurement of Coefficient of Friction Using / Tatiana Kelemenová, Miroslav Dovica, Pavol Božek, Ivana Koláriková, Ivana Koláriková, Ondrej Benedik ,Ivan Virgala,Erik Prada,Ľubica Miková ,Tomáš Kot and Michal Kelemen // Variable Incidence Tribometer. Symmetry. – 2020. – 12(8), 1235 p. https://doi.org/10.3390/sym12081235
12. Kelemenová, T. Specific problems in the measurement of coefficient of friction in sliding pairs / T. Kelemenová, Z. Tkáč, and M. Varga // Acta Mech. Slovaca. – 2020. – Vol. 24. – No. 3, P. 30–35,
13. Zhang, X. Дослідження характеристик зношування та тертя у фрикційних парах гідравлічного насоса високого тиску / Zhang, X., Li, B., & Wang, X. // Tribology International. – 2022. – 167 p. https://doi.org/10.1016/j.triboint.2022.107421
14. Bian, B. Lubrication, friction, and wear characteristics of textured slipper pairs in axial piston pumps. Lubricants. 2024. – 12(11), 370 p. https://doi.org/10.3390/lubricants12110370
15. He, Y., Tan, M., & Lu, H. Kinetostatic modeling and analysis of compliant mechanisms. Meccanica. 2022. – 57, 991–1012p. https://doi.org/10.1007/s11012-021-01386-7